years unexpected applications, for example in the theory of Riemann's Zeta-function.

VI. *Algebraic and transcendental numbers* (24 pp.): Besides the fundamental definitions and theorems (including the proof that e and π are transcendental numbers) a certain class of transcendental numbers which have been called by Maillet* *Liouville numbers* are studied. Since Maillet introduced the name and made in his book a systematic (although somewhat obscure) study of these numbers and since Perron in another work (*Die Lehre von den Kettenbrüchen*, Leipzig, 1913) gives full credit to Maillet, it is obviously an oversight that Maillet's book is not mentioned in the *Irrationalzahlen*.

The literature references are arranged for each chapter separately and seem fairly complete. However, Minkowski, *Diophantische Approximationen*, Leipzig, 1907, is not quoted. Borel, *Leçons sur la Théorie de la Croissance*, Paris, 1910, pp. 118–168, might have been mentioned in connection with chapters V, VI. It is not very satisfactory that even in the case of large books no page reference is given; a reference such as: L. Euler, *Introductio in Analysin infinitorum*, I, 1748 (a book of over 300 quarto pages, in Latin), is perhaps not easily run down.

The only American author referred to seems to be Huntington (TRANSACTIONS OF THIS SOCIETY, vol. 6 (1905).

A. J. KEMPNER

Grundlehren der Neueren Zahlentheorie. By Paul Bachmann. Volume III of Göschen's Lehrbücherei, Gruppe I, Reine Mathematik. Berlin und Leipzig, Vereinigung Wissenschaftlicher Verleger, 1921. xi + 252 pp.

The book under discussion represents the second edition of volume LIII of the well known Sammlung Schubert, G. J. Göschensche Verlagshandlung, Leipzig, 1907. No important changes have been made. New is a chronological table of the known proofs of the famous *Law of Quadratic Reciprocity:* fifty-six proofs from the year 1796 (Gauss' first proof, published 1801) to Frobenius' modification of Zeller's proof, 1914.† The mathematical basis of each proof is indicated; in thirty-two cases Gauss' lemma or a variant of Gauss' lemma is given as the foundation. A five-page alphabetic index has also been added.

Since the first edition was reviewed by J. W. Young in this BULLETIN (vol. 15 (1908-9), pp. 463-5), it is not necessary to consider in detail the mathematical contents of this excellent little book. The small corrections which were suggested in this review have been carried out.

The new edition is posthumous; it contains a five-page necrology, *Zum Gedächtnisse von Paul Bachmann*, by Robert Haussner. Bachmann's influence in stimulating interest in the theory of numbers has been so great that American readers may be interested in a few notes concerning his life and his work.

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* Maillet, *Introduction à la Théorie des Nombres Transcendants*, Paris, 1906, particularly chapters II, III.

† A less complete table is contained in Bachmann's *Niedere Zahlentheorie*, vol. I, p. 203 ff.

Paul Bachmann, born Berlin, June 22, 1837; studied mathematics 1855–1862 at Göttingen and Berlin (at Göttingen, together with Dedekind, under Dirichlet); Privatdozent at Breslau, 1864; ausserordentlicher Professor, Breslau, 1867; Professor at the Akademische Lehranstalt, Münster i. W., 1873; retired 1890; did not occupy an official position from this time on until his death, March 31, 1920.

Bachmann's influence is based, not so much on his original contributions to mathematics,* as on his series of scientific textbooks on the theory of numbers. It was his ambition to cover in these books the total field of this theory.

The *Gesamtdarstellung* included, up to the time of Bachmann's death, the following volumes:

I. *Elemente der Zahlentheorie*, 1892,
II. *Analytische Zahlentheorie*, 1894,
III. *Lehre von der Kreisteilung*, 1872,
IV (1). *Arithmetik der quadratischen Formen*, 1898,
IV (2). (*Über Reduktion der Formen*),
V. *Arithmetik der Zahlenkörper*, 1905.

Of these, IV (2) was finished in manuscript in 1915, but could not, on account of economic conditions in Germany, be published, although "nach dem übereinstimmenden Urteile aller Mathematiker, die das Manuscript kennen gelernt haben, ein ganz ausgezeichnetes Werk vorliegt." For a second volume of V, *Über spezielle Zahlenkörper*, only preliminary work was carried out.

Besides the *Gesamtdarstellung*, Bachmann has published several books:
Vorlesungen über die Natur der Irrationalzahlen, 1892,
Niedere Zahlentheorie, vol. I, 1902,
Niedere Zahlentheorie, vol. II, 1910 (Additive Zahlentheorie),
Grundlehren der Neueren Zahlentheorie, 1907 (2d ed. 1921),
Das Fermatproblem in seiner bisherigen Entwicklung, 1919.

The difficulty of carrying out the program outlined by this list will be realized by every mathematician familiar with the theory of numbers. With admirable skill Bachmann brings out underlying unifying principles and, even in such fields as the additive theory of numbers, traces the threads connecting a large number of apparently disconnected results.

Since many of the books—for example those dealing with the analytic theory and with the additive theory—were written at a period when these branches of mathematics were in a stage of vigorous development, it is inevitable that some of the volumes no longer adequately represent the complete theory; but for a long time to come it will be safe to advise any student interested in the theory of numbers to study carefully Bachmann's works. Perhaps the highest praise one can give these books is to emphasize that the progress in mathematics which has made a few of them appear slightly antiquated is probably in no small measure due to the interest aroused by them.

A. J. KEMPNER

* This statement should not be interpreted as a criticism of the value of his original contributions.