and since the expressions for the B's in terms of A's are similar to (7), we have reduced the problem to one of lower order.

To complete our proof, we reduce the case for $m + 1$ subscripts to that for m. We first obtain values for the A's not involving one subscript, say k, from the equations and conditions not involving k, by the case assumed as the basis of the induction. Then we make the substitutions:

$$(12) \qquad B_{i_1 \ldots i_n k} = B'_{i_1 \ldots i_n} + (-1)^n \frac{\partial A_{i_1 \ldots i_n}}{\partial x_k},$$

which effects the desired reduction.

This proves the sufficiency of the conditions; that they are necessary follows by direct substitution. A more complete discussion of the properties of multiple integrals is given in an expository article that will appear in the ANNALS OF MATHEMATICS.

HARVARD UNIVERSITY

KIRKMAN PARADES*

BY F. N. COLE

On examining the complete list of non-equivalent triad systems in 15 letters published in the MEMOIRS OF THE NATIONAL ACADEMY OF SCIENCES (vol. 14, No. 2, pp. 77–80), it appears that only four of these are Kirkman systems, if this name be applied to those cases where the 35 triads divide into seven sets (or columns) of five with each column containing all the 15 letters. Such a seven-column arrangement might be called a Kirkman parade. And it turns out that three of the Kirkman systems give each two non-equivalent parades, while the fourth system gives only one parade.

Kirkman proposed his problem in the LADY'S AND GENTLEMAN'S DIARY for 1850. The seven solutions were correctly given by Woolhouse in the same Diary for 1862 and 1863. In 1881 Carpmael published a list of eleven solutions in the PROCEEDINGS OF THE LONDON MATHEMATICAL SOCIETY (vol. 12, pp. 148–156). But his sixth and seventh items duplicate the third and fourth, and the fifth and eleventh duplicate the ninth and tenth.

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In forming the columns of a parade, any two of them can always be reduced to one of the types

	1	2	3	1	4	7		1	2	3	1	4	7
	4	5	6	2	5	8		4	5	6	2	5	10
(α)	7	8	9	3	10	13	(β)	7	8	9	3	8	13
	10	11	12	6	11	14		10	11	12	6	11	14
	13	14	15	9	12	15		13	14	15	9	12	15

We may say that two columns are *laced* in the mode (α) or the mode (β), and the seven parades are readily distinguished by this lacing:. The parades follow in tabular form, together with the substitution group of each system and parade and the scheme of interlacing of the columns.

I–II

1	2	3	1	4	7	1	5	15	1	9	13	1	6	10	1	8	11	1	12	14
4	5	6	2	5	8	2	9	10	2	4	12	2	11	15	2	7	14	2	6	13
7	8	9	3	10	13	3	4	14	3	5	11	3	7	12	3	6	9	3	8	15
10	11	12	6	11	14	6	8	12	6	7	15	4	8	13	4	10	15	4	9	11
13	14	15	9	12	15	7	11	13	8	10	14	5	9	14	5	12	13	5	7	10
												1	6	10	1	8	11	1	12	14
												2	7	14	2	6	13	2	11	15
	"			"			"			"		3	8	15	3	7	12	3	6	9
												4	9	11	4	10	15	4	8	13
												5	12	13	5	9	14	5	7	10

These two parades are included in a single triad system, which contains 120 conjugates of each of them. The group of the system is of order $8!/2$. The group of I is of order 168 and is generated by (1 11 7 5 9 12 2)(3 8 13 10 14 15 4) and (1 13 12 11 15 3 4)(2 8 14 7 9 5 10); it is transitive in all the letters but 6. The group of II is also of order 168, but it is transitive in seven and in eight letters, being generated by (1 2 9 13 6 3 10)(4 14 8 11 15 12 5) and (1 10 13 9 2 6 3)(4 8 7 14 11 5 15).

In both I and II the columns have only the lacing (α).

III–IV

1	2	3	1	4	7	1	5	15	1	9	10	1	6	13	1	8	11	1	12	14
4	5	6	2	5	8	2	9	13	2	4	12	2	7	14	2	6	10	2	11	15
7	8	9	3	10	13	3	4	11	3	5	14	3	8	12	3	7	15	3	6	9
10	11	12	6	11	14	6	7	12	6	8	15	4	10	15	4	9	14	4	8	13
13	14	15	9	12	15	8	10	14	7	11	13	5	9	11	5	12	13	5	7	10
												1	6	13	1	12	14	1	8	11
												2	11	15	2	6	10	2	7	14
	"			"			"			"		3	8	12	3	7	15	3	6	9
												4	9	14	4	8	13	4	10	15
												5	7	10	5	9	11	5	12	13

These are included in a single triad system, which contains 12 conjugates of each of them. The group of the system is of order 288. The group of III is of order 24 and is generated by (1 8 5)(2 15 11)(3 6 9)(4 10 12)(7 14 13) and (1 10 2 13)(5 15 4 12)(7 11 8 14)(6 9). The group of IV is also of order 24 and is generated by (1 10 13)(3 6 9)(4 5 11)(8 15 12) and (1 12 2 15)(4 11 5 14)(7 10 8 13)(3 9). The last column in each case is in relation (α) to all the other columns; each of the latter has two (α) lacings and four (β) lacings.

V–VI

1	2	3	1	4	7	1	5	13	1	6	8	1	9	10	1	11	15	1	12	14
4	5	6	2	5	10	2	4	12	2	11	13	2	7	14	2	6	9	2	8	15
7	8	9	3	8	13	3	9	11	3	7	12	3	5	15	3	4	14	3	6	10
10	11	12	6	11	14	6	7	15	4	10	15	4	8	11	5	8	12	4	9	13
13	14	15	9	12	15	8	10	14	5	9	14	6	12	13	7	10	13	5	7	11
												1	9	10	1	11	15	1	12	14
												2	8	15	2	7	14	2	6	9
	"			"			"			"		3	4	14	3	6	10	3	5	15
												5	7	11	4	9	13	4	8	11
												6	12	13	5	8	12	7	10	13

These again are both contained in one triad system. They have both the same group as the system, viz., the tetrahedral group generated by (1 12)(2 7)(5 15)(8 11)(6 10)(9 13) and (1 2 3)(4 12 7)(6 10 9)(5 11 8). The last three columns of each have the lacing (α); the other lacings are all of type (β).

VII

1	2	3	1	4	7	1	5	9	1	6	15	1	8	11	1	10	13	1	12	14
4	5	6	2	5	10	2	4	14	2	9	11	2	6	13	2	7	12	2	8	15
7	8	9	3	8	13	3	10	15	3	7	14	3	4	12	3	6	9	3	5	11
10	11	12	6	11	14	6	8	12	4	8	10	5	7	15	4	11	15	6	7	10
13	14	15	9	12	15	7	11	13	5	12	13	9	10	14	5	8	14	4	9	13

This has a group of order 21, generated by (1 2 12 3 7 4 14)(5 8 11 9 15 10 6) and (2 7 12)(3 4 14)(5 6 11)(8 9 15), which is also the group of the triad system. The lacings of the columns are all of type (β).

COLUMBIA UNIVERSITY