

SHORTER NOTICES

Oeuvres de G. H. Halphen. Publiées par les soins de C. Jordan, H. Poincaré, É. Picard, avec la collaboration de E. Vessiot. Tome III.* Paris, Gauthier-Villars, 1921. xii + 518 pp.

The Paris Academy of Sciences proposed as the subject of the grand prize in mathematics for 1880 the following question: "To perfect in some important part the theory of ordinary linear differential equations." Halphen submitted a memoir in which he proposed to determine those ordinary linear differential equations which are reducible to integrable forms by means of transformations of a certain class; and the prize was awarded to this memoir. The text of Hermite's report on the memoir is reproduced in an article on Halphen published by C. Jordan in LIOUVILLE'S JOURNAL in 1889 and reprinted on pages v to xii of the volume under review. (See also p. 33 of vol. I of Halphen's *Oeuvres*.)

Halphen formulated in the following manner the exact "double question" which he treated:

(1) Having given a linear differential equation with variable X and unknown Y , to determine if there exists a substitution

$$x = \varphi(X), \quad y = Y\psi(X),$$

such that, x and y being taken for the new variable and the new unknown, the transformed equation belongs to one of the following three categories:

- I. Equations with constant coefficients;
- II. Equations whose general integral is rational;
- III. Equations with doubly periodic coefficients with the same periods and with a uniform general integral.

(2) Having determined the existence of the substitution, to effect the integration.

The memoir in which he treats this question is reprinted on pages 1-260 of the volume under review. It opens with an excellent fourteen-page summary of the results attained. The response to the first question is entirely decisive: necessary and sufficient conditions are obtained (and summarized in the introduction) for the existence of the substitution named of such sort that one may apply them to a particular equation by means of straightforward reckoning. The propositions relating to the second part are not quite so definitive; but they are far-reaching enough to have important applications, particularly to equations of the second order.

The judgment of the Academy, awarding the grand prize to Halphen for this memoir, was scarcely rendered when the Berlin Academy proposed for the Steiner prize "the solution of an important question in the theory of twisted algebraic curves." In 1870 Halphen had published a short note on the classification of these curves. He now returned to this subject and produced his classic memoir on the classification of twisted algebraic curves (reprinted on pp. 261-455 of the volume under review). It was crowned conjointly with a memoir by Noether submitted on the same

* For a short review of volumes I and II see this BULLETIN, vol. 27 (1920-21), pp. 466-468.

occasion. By many this is esteemed to be Halphen's chief work. Some account of it is given in the review of volumes I and II in connection with Poincaré's analysis of the memoir on pages xxx-xxxi of volume I.

Besides these two master memoirs, volume III of the *Oeuvres* contains the introductory article by C. Jordan already referred to, and two articles by Halphen closely related to the former of the two already treated. They both deal with linear differential equations of the fourth order: one is a brief note (pp. 457-461) from the *COMPTES RENDUS* and the other is a longer article (pp. 463-514) from the *ACTA MATHEMATICA* on the invariants of linear differential equations of the fourth order.

The four articles reprinted in this volume may justly be said to contain two great contributions by Halphen. The one is the memoir on twisted algebraic curves and the other is the work on linear differential equations contained in the first, third and fourth papers. These are all entirely inspired by the theory of differential invariants (to which Halphen had previously made characteristic contributions); and they are treated together by Halphen himself (vol. I, pp. 32-35) in the summary of his work prepared by him on the occasion of his candidacy before the Paris Academy in 1885.

R. D. CARMICHAEL

Graphical and Mechanical Computation. By Joseph Lipka. New York, John Wiley and Sons, 1918. ix + 264 pp.

This book is something new in American text-books. It treats two subjects of vital importance to engineering students that heretofore have found little place in the curricula of American engineering schools. The first part of the book is devoted to the graphic solution of engineering problems by means of networks of scales, various kinds of coordinate paper, the charting of equations in three variables and more particularly by means of alignment charts. The second half of the book is concerned with the question of empirical formulas for both non-periodic and periodic curves giving numerical, graphic and mechanical methods for determining the constants. For data which cannot be fitted to convenient formulas, numerical, graphic and mechanical methods are developed in the last two chapters for interpolation, differentiation and integration.

The book is rich in illustrations and applications to practical engineering problems, showing clearly the value of the graphic methods in reducing the drudgery of long computations. The three chapters on alignment charts are recommended to the student who wishes to know something of the nomographic charts of Professor M. d'Ocagne. The work is also published in two volumes.

A. R. CRATHORNE

Funktionentheorie. By Konrad Knopp. 2d edition. Berlin and Leipzig, Vereinigung wissenschaftlicher Verleger. Part I, 1918, 140 pp. Part II, 1920, 138 pp.

This "Funktionentheorie" is a completely revised edition of the work that appeared under the same title in 1913 in the *Sammlung Göschen*. It is issued in a form similar to the old one by the *Vereinigung wissenschaftlicher Verleger* which, since the war, has continued the publishing