

numbers less than 100,000 which are prime to 2 and 5. The inclusion of the smallest factor 3 adds little to the value of the table and much to the bulk. It is decidedly inferior to Burckhardt's table in convenience of arrangement, and is approximately twice as bulky as a table which omits multiples of 2, 3, 5 and 7. It is difficult to see what end is served by the republication of such a table at the present time.

The book also contains a table of "Tessaréen" numbers, which seem to be prime numbers of the form  $a^2 - a - 1$ . The connection between such numbers and the numbers representable by the binary quadratic form  $x^2 - xy - y^2$  is not indicated. A list of such numbers with the corresponding values of  $a$  is given, the list extending as far as the prime 19991. What particular use is to be made of this table is not indicated. The "preliminary explanation" avoids giving any demonstration of the properties of "Tessaréen" numbers, on the ground that such a demonstration would not teach anything to one who was already familiar with the theory of numbers, and would not be understood by one who was not.

The author of the preliminary explanation—name not signed, but perhaps Lebon—deplores the neglect of the theory of numbers, which neglect he attributes to the vicious methods and barbarous terminology of German mathematicians. Gauss especially comes in for a thorough castigation for his bizarre and incoherent formulas!

Five pages of the introduction are devoted to a biography of Inghirami by Giovanni Giovannozzi. Inghirami's most important work seems to have been done in astronomy and in geodesy. The factor tables here republished appeared for the first time in 1832 at the end of a volume on *Elementi di Matematiche*. He evidently did not know of Burckhardt's tables published some twenty years previously.

D. N. LEHMER.

*Lectures of the Theory of Plane Curves.* By SURENDRAMOCHAN GANGULI, M.Sc., Lecturer in Pure Mathematics, University of Calcutta. Part I, x + 140 pp.; Part II, xiii + 350 pp., and 13 pages of figures. Published by the University of Calcutta, 1919.

THESE lectures were delivered to postgraduate students and comprise in a fairly satisfactory manner most of the topics usually presented in an elementary course on plane curves.

The first part is concerned with the general theory, the second with cubics and quartics. In teaching the subject, Ganguli had constant recourse to the classic treatises of Salmon and Clebsch, and the works of Basset, Scott, and others, and laid particular stress on Sylvester's theory of residuation. But instead of using this theory for the systematic study of the geometry of point groups on curves, Ganguli merely makes occasional applications of it. The definition of a curve based upon the representation of a function, in §§ 10-11, is confusing and inadequate. In the first place there is no proper postulation of the space in which curves are to be defined. Metric and projective concepts are indiscriminately mixed up, so that, as a consequence, a proper formulation of the systems of coordinates is impossible. From the present prevailing critical point of view this is one of the weakest points of Ganguli's lectures. What, for example, do the following statements mean, when the graph of a function is not defined? "In the modern theory of functions, it is held that a function can be completely defined by means of a graph arbitrarily drawn in the finite and continuous domain of the independent variable." Again, notice the erroneous declaration, "The modern theory of functions says that the equation  $F(x, y) = 0$  cannot in general represent a curve, it can do so if  $y$  can be expressed as a regular (or rational) function  $f(x)$  of  $x$ , i.e., if  $f(x)$  is a continuous, finite and differentiable and separately monotonous function. It is only by a combination of these conditions that  $y = f(x)$  can represent a curve." From this it is evident that the author still relies on the hazy notions about a curve as they were held at the time of Euler. Not the slightest attention is given to complex domains, or to what kind of an equation  $F(x, y) = 0$  is.

The principle of duality and the corresponding use of line coordinates are brought in incidentally and without an effort at systematic representation. Using the designation "reciprocal curves" suggests the idea of the special duality involved in polar reciprocity. Moreover the lack of a proper projective space makes an adequate treatment of the behavior of curves at infinity impossible.

The typography of both volumes could be considerably improved in an eventual future edition. Much more attention should also be given to the reading of proofs.

But aside from the defects mentioned above the beginner

may learn a good deal about the properties of algebraic curves, so that in this respect the publication of a new English treatise on curves is not without value, and deserves commendation.

ARNOLD EMCH.

*Nouvelles Méthodes de Résolution des Equations du 3e Degré.* By le VTE. DE GALEMBERT. Paris, Vuibert, 1919. 22 pp.

This pamphlet gives a method for rapid numerical calculation of the real roots of the cubic equation

$$x^3 + Ax^2 + Bx + C = 0,$$

new as far as I am aware, for the case in which there are three real roots. The equation is reduced to the form  $z^3 - 3z = y$ ; the latter equation has three real roots if  $|y| \leq 2$ . A table of corresponding values of  $z$  and  $y$  is computed once for all, by means of which the values of  $z$  may be found accurately to two decimals, whenever  $y$  is known to six places. An additional table gives  $z$  and  $y$  in terms of  $x$ ,  $A$ ,  $B$  and  $C$ , for the different combinations of signs which the coefficients may have in the general equation. The actual calculation of the roots is very much simplified in this way.

A. DRESDEN.

*The Integral Calculus.* By JAMES BALLANTYNE. Boston, Published by the author, 1919. 41 pp.

THE subtitle of this small book is sufficiently descriptive of its scope. It reads, "On the integration of the powers of transcendental functions, new methods and theorems, calculation of Bernoullian numbers, rectification of the logarithmic curve, integration of logarithmic binomials, etc."

The author has several new series expansions of transcendental functions; but does not burden his tale with arguments as to the rigor of his methods or the general validity of his formal results. The book gives the impression of having been written for the fun of it, by a very ingenious gentleman, who was having a fine time giving free rein to his analytical processes and going gladly wherever those steeds—dangerous if unchecked—might lead him.

The style of the text may be indicated by such expressions of olden time flavor as, "integrals of even powers of  $\sin x dx$  to radius 1"; "the value of  $C$  is the area of the full quadrant