brought to light a new world of thought as to the relations of space and time to the ultimate data of perceptual knowledge. "The present work is largely concerned with providing a physical basis for the more modern views which have thus emerged. The whole investigation is based on the principle that the scientific concepts of space and time are the first outcome of the simplest generalizations from experience, and that they are not to be looked for at the tail end of a welter of differential equations."

Three main streams of thought—the scientific, the mathematical, and the philosophical—are relevant to the theme of the enquiry. About half the book is given to parts I and II on the traditions of science and the data of science respectively. In part III on the method of extensive abstraction we have a philosophical and postulational treatment of the space-time manifold; and this is employed in part IV to yield a theory of objects. The fundamental assumption elaborated "is that the ultimate facts of nature, in terms of which all physical and biological explanation must be expressed, are events connected by their spatio-temporal relations, and that these relations are in the main reducible to the property of events that they contain (or extend over) other events which are parts of them."

R. D. CARMICHAEL.

Differential Equations. By H. BATEMAN. Longmans, Green and Company, London and New York, 1918. xii + 306 pp.

"THE subject of Differential Equations has grown so rapidly in recent years that it is difficult to do justice to all branches of the subject in a single volume." So reads the opening sentence of the preface to the book under review. The statement is literally true; but it is nevertheless of such form that its connotation may be misleading to the learner. One who is not acquainted with the subject of differential equations may justly conclude from this sentence that it is possible, though indeed difficult, to do justice to all branches of the subject in a single volume. And yet it is probable that no one who knows the field would be willing to maintain such a judgment. It is in fact true that it would be difficult to do justice to all branches of the subject in ten volumes of the size of the one under consideration.

In the second sentence of his preface the author indicates

the purpose of his book in the following language: "In writing this book I have endeavored to supply some elementary material suitable for the needs of students who are studying the subject for the first time, and also some more advanced work which may be useful to men who are interested more in physical mathematics than in the developments of differential geometry and the theory of functions."

From this it will be seen that the author in his single exposition seeks to serve two rather different purposes. In so far as these purposes diverge there is an evident possibility that the provision for one will interfere with the best provision for the other. It seems to the reviewer, in fact, that the more advanced matter has been provided in such way as to interfere with the usefulness of the book to students who are studying the subject for the first time. There does not appear to be a sufficient indication (even with the help of the preface) as to what parts of the book are best suited to the needs of the beginner. It is not clearly apparent, indeed, that the author himself had in mind a definite separation of material for serving the two purposes named. Moreover, the exposition from time to time assumes on the part of the reader a range of mathematical knowledge which is not the common possession of students pursuing the subject for the first time. A similar statement may also be made relative to those who are interested primarily in physical mathematics.

These considerations may lead to the fear that the usefulness of the book has not been increased by the attempt to serve simultaneously two distinct purposes which are not so intimately bound together as to justify this plan of treatment without a more distinct separation and indication of parts to serve these purposes.

Occasionally the rhetorical relations within a sentence might be improved. Greater care in proofreading would have resulted in the avoidance of several minor blemishes. But so far as the reviewer has observed these defects are not likely to cause the reader serious inconvenience.

Having said these things, let us turn now to matters which it is more pleasant to dwell upon. Even a rapid examination of the contents of the volume will bring out the fact that it contains a considerable amount of useful material not easily accessible elsewhere, and indeed some new material in Chapters VII and VIII.

Chapter II (pages 17–59) on integrating factors contains a rather wide range of useful and interesting matter, a part of which one would hardly expect to find under the title given. Further elementary methods of solution are treated in Chapter III (pages 60–86) under the heading transformations. In Chapter V (pages 103–115) we have a brief treatment of differential equations with particular solutions of a specified type. Some rather interesting results are obtained by starting from this elementary point of view, results which it would probably be more difficult to secure by any other method. They have to do especially with the algebraic integrals of certain important equations. Chapters VI to XI bear the following titles: partial differential equations, total differential equations, partial differential equations of the second order, integration in series, the solution of linear differential equations by means of definite integrals, the mechanical integration of differential equations.

Chapter VIII (pages 169–222) on partial differential equaions of the second order will be found particularly rich in material with an appeal to those who are interested primarily in physical mathematics. Here one finds a considerable treatment of the equation of wave propagation in three dimensions, of the Maxwell equations, of the electron equations, of Laplace's equation, and of the equation of the conduction of heat.

The chapter on solutions by means of definite integrals contains a useful summary of material, a part of which is not so conveniently found elsewhere so far as the reviewer is aware.

Those who give instruction to elementary classes in differential equations will find in this book of Bateman's a useful source of supplementary material of the nature both of additional topics for special report by members of the class and of problems to furnish a variation from those in the basic text. Some of the problems here included are rather unusual in character (as for instance 2 and 6 and 17 of the miscellaneous examples) and hence are of value to the instructor who wishes to introduce greater variety.

R. D. CARMICHAEL.