Not all equations of that form are reducible to the form (10); for example

$$\Delta f'(x) + 2f'(x) + c(x)f(x) = 0$$

is not. However, equations with equal invariants which are not self-adjoint do not seem to be of great interest.

From the preceding theorem it follows that *if a self-adjoint equation of form* (10) *is of finite rank with respect to one of the transformations* (S) *or* (T), *it is of the same rank with respect to the other.*

Using the formula*

$$I_{S_n}(x) = I(x) + \sum_1^{n-1} [I(x+k) - J(x+k)]$$
$$- \Delta \frac{d}{dx} \log \left[\prod_{k=0}^{n-1} I(x+k) \prod_{k=0}^{n-2} I_{S_1}(x+k) \cdots I_{S_{n-1}}(x) \right]$$

and noting that for the self-adjoint case

$$I(x) = J(x) = - c(x),$$

we have

$$- c(x) = \Delta \frac{d}{dx} \log \left[\prod_{k=0}^{n-1} (- c(x+k)) \prod_{k=0}^{n-2} I_{S_1}(x+k) \cdots I_{S_{n-1}}(x) \right]$$

as a necessary and sufficient condition that the self-adjoint equation (10) *be of rank* $n+1$ *with respect to each of the transformations* (S) *and* (T).

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THE SECOND VOLUME OF VEBLEN AND YOUNG'S PROJECTIVE GEOMETRY.

Projective Geometry. By OSWALD VEBLEN and J. W. YOUNG. Boston, Ginn and Company; Vol. 2, by Oswald Veblen, 1918. 12 + 511 pages.

IN volume I, Veblen and Young were concerned particularly with those theorems of projective geometry which can be proved on the basis of their assumptions A of alignment, assumptions E of extension, and an assumption P of pro-

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* Formula (8) in my thesis, l.c.

jectivity. In some cases use was also made of the assumption H_0 that the diagonal points of a complete quadrangle are non-collinear. A space satisfying A and E is called a general projective space. This term does not seem to the reviewer to be altogether appropriate. According to this terminology real projective spaces, complex projective spaces, etc., are all general projective spaces. This seems rather like saying that every special projective space is a general projective space. It would seem in some respects preferable to omit the word general and say simply that every space satisfying A and E is a projective space.*

In volume II various special projective spaces are studied, each special projective space being a space in which, in addition to A and E, certain other postulates are satisfied. One example of a special projective space is a proper projective space, a space satisfying A, E and P. A projective space satisfying the assumption H that if any harmonic sequence exists not every one contains only a finite number of points is called a non-modular projective space. A modular projective space is one in which every harmonic sequence contains only a finite number of points.

Assumptions A, E, P and H_0 hold true in both the ordinary real and the ordinary complex projective spaces of three dimensions. A space satisfying these assumptions is not necessarily continuous in the sense of Dedekind. Indeed it does not necessarily possess even pseudo-archimedean continuity. Veblen and Young's assumptions A, E and H_0 correspond rather closely to the first fourteen of a set of nineteen postulates published by Pieri in 1899.† The following is a rough translation of Pieri's set.

POSTULATE I°. *Projective points form a class.*

POSTULATE II°. *There exists at least one projective point.*

POSTULATE III°. *If there exists one projective point, there exists at least one other one.*

POSTULATES IV° AND V°.‡ *If a, b are projective points $(a \neq b)$, the line ab is a class of points.*

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* The same objections would not apply to the use of the term general projective geometry to designate the totality of all theorems that can be proved on the basis of A and E. There is only one such body of theorems and therefore only one such geometry, while there are many sorts of spaces satisfying A and E.

† M. Pieri, "I principii della geometria di posizione composti in sistema logico deduttivo," *Memorie della Reale Accademia delle Scienze di Torino*, vol. 48 (1899), pp. 1–62.

‡ Cf. A_1.

POSTULATE VI°.   *If a and b are distinct projective points, the line ab is contained in the line ba.*

POSTULATE VII°.   *If a and b are distinct projective points, a belongs to the line ab.*

POSTULATE VIII°.*   *If a and b are distinct projective points, the line ab contains at least one point distinct from a and from b.*

POSTULATE IX°.   *If a and b are distinct projective points and c is a point of the line ab distinct from a, then b belongs to the line ac.*

POSTULATE X°.   *Under the same hypothesis ab contains ac.*

POSTULATE XI°.†   *If a ≠ b, there exists at least one point not on the line ab.*

POSTULATE XII°.‡   *If a, b, c are non-collinear projective points and a′ is a point of bc distinct from b and from c and b′ is a point of ac distinct from a and from c, then the line aa′ has a point in common with the line bb′.*

POSTULATE XIII°.§   *If a, b, c are non-collinear points, there exists a point not in the plane abc.*

POSTULATE XIV°.∥   *If a ≠ b and c is a point of ab distinct from a and from b, then the fourth harmonic of c with respect to a and b is distinct from c.*

POSTULATE XV°.   *If a, b, c are distinct points of a line r and d is a point of the line r not belonging to the segment¶ (abc) nor coinciding with a or with c, then d belongs to the segment (bca).*

POSTULATE XVI°.   *If a, b, c are distinct collinear points of a line r and a point d belongs to both of the segments (bca) and (cab), then it does not belong to the segment (abc).*

POSTULATE XVII°.   *If a, b, c are distinct collinear points and d is a point distinct from b on the segment (abc) and e is a point of the segment (adc), then the point e is on the segment (abc).*

Postulate XVIII° is a form of Dedekind cut postulate as applied to the points of a segment.

POSTULATE XIX°.**   *If a, b, c, d are distinct non-coplanar*

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* Cf. Veblen and Young's E_0.
† Cf. Veblen and Young's E_2.
‡ Cf. Veblen and Young's A_3.
§ Cf. E_3.
∥ Cf. Assumption H_0. Pieri points out that in the presence of Postulates VI°, XII° and XIII°, Postulate XIV° is equivalent to the proposition that the diagonal points of a complete quadrangle are collinear.
¶ The *segment* (abc) is defined as the set of all points [x] such that, for some pair of points y and z on the line abc which are harmonic conjugates of each other with respect to a and c, x is the fourth harmonic of b with respect to y and z.
** Cf. Assumption $E_3′$.

points, then for every point e not in any one of the planes abc, abd, acd, bcd there exists at least one point common to the figures ae, bcd.

It seems not far from accurate to say that Veblen and Young's set A, E, H_0 differs from Pieri's set $I°$–$XIV°$, $XIX°$ chiefly in that Veblen and Young postulate simply the four assumptions A_1, A_2, E_0 and E_1 instead of Pieri's I–X, which latter may be (perhaps rather roughly) considered as corresponding to the result of breaking up (or analyzing) the group of assumptions A_1, A_2, E_0 and E_1, in a certain fashion, into a larger number of weaker postulates. Every space that satisfies A, E, H_0 satisfies also $I°$–$XIV°$ and conversely.

Veblen secures a categorical set of postulates for real projective space by defining the notion of order on a net of rationality and the notion of open cuts in such a net and adding, to A and E, assumption H and the following assumptions C and R.

ASSUMPTION C. *If every net of rationality contains an infinity of points, then on one line l in one net R $(H_0H_1H_\infty)$ there is associated with every open cut (A, B) with respect to the scale H_0, H_1, H_∞, a point $P_{(A, B)}$ which is on l and such that the following conditions are satisfied:*

(1) *If two open cuts (A, B) and (C, D) are distinct, the points $P_{(A, B)}$ and $P_{(C, D)}$ are distinct;*

(2) *If (A_1, A_2) and (B_1, B_2) are any two cuts and (C_1, C_2) any open cut between two points A and B of R $(H_0H_1H_\infty)$ and if T is a projectivity such that*

$$T(H_\infty AB) = H_\infty P_{(A_1, A_2)} P_{(B_1, B_2)},$$

then $T(P_{(C_1, C_2)})$ is a point associated with some cut (D_1, D_2) between (A_1, A_2) and (B_1, B_2).

ASSUMPTION R. *On at least one line, if there is one there is not more than one chain.*

Pieri secures the same result by defining the notion of a segment and adding postulates $XV°$–$XVIII°$.

It seems to the reviewer that one might naturally feel, in view of the complicated nature of Veblen's assumption C, that Pieri's $XV°$–$XVIII°$ constitute a simpler group of assumptions than that constituted by Veblen's H, C and R. Veblen's set has an apparent advantage in that his assumption C holds true for a complex space as well as for a real one, while this is not true of Pieri's postulate $XV°$. But let us consider

a second article by Pieri, published in 1905.* In this article
Pieri gives a set of postulates for complex projective geometry
of infinitely many dimensions. The first 10 of these corre-
spond closely to the first 14 of his first set except for the fact
that (a) instead of XI° and XIII° of set I we have† in set II
a postulate to the effect that for every complex space S_n there
exists at least one point not in S_n and (b) postulate XIV° is
replaced by a postulate to the effect that the diagonal points
of a complete quadrangle are non-collinear.‡

In postulate XII and in most of the remaining postulates
XII–XXX use is made of a new undefined notion, the notion
of a chain. Postulates XII–XIX are (freely translated) as
follows:

POSTULATE XII. *If a, b, c are distinct points of a complex
line, the chain of the points a, b, c—indicated by the symbol
$|abc|$—is a class of points belonging to that line.*

POSTULATES XIII AND XIV. *If a, b, c are distinct collinear
complex points, the chain $|abc|$ is contained in each of the
chains $|acb|$ and $|bac|$.*

POSTULATE XV. *Under the same hypothesis a belongs to
the chain $|abc|$.*

POSTULATE XVI. *If a, b, c, d are distinct collinear points
and a', b', c', d' are distinct collinear points in perspective corres-
pondence with a, b, c, d, then d belongs to $|abc|$ if and only if d'
belongs to $|a'b'c'|$.*

POSTULATE XVII. *If a, b, c are three distinct collinear points,
the fourth harmonic of c with respect to a and b belongs to the chain
$|abc|$.*

POSTULATES XVIII AND XIX. *If a, b, c, d are four distinct
collinear points and d belongs to the chain $|abc|$, then c belongs
to $|abd|$ and $|abd|$ is contained in $|abc|$.*

If in the definition of "segment (abc)" quoted above for
real space the phrase "line abc" is replaced by "chain $|abc|$"
the resulting definition holds good for complex space.

Postulate XX of set II is equivalent to the proposition
obtained by substituting "chain $|abc|$" for the second "line
r" in the statement of postulate XV° of set I. Postulates
XXI and XXII of set II are respectively equivalent to XVI°
and XVII° of set I.

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* "Nuovi principii di geometria projettiva complessa," *Memorie della
Reale Accademia delle Scienze di Torino*, vol. 55 (1905), pp. 189–235.
† A natural change to make in passing from a set of postulates for
three dimensions to one for infinitely many dimensions.
‡ Cf. Veblen and Young's H_0.

The set of postulates I–XXII hold true in real as well as in complex space.   If, in this set, postulate X is replaced by XI°, XIII° and XIX° of set I, and Veblen's R and a suitable Dedekind cut postulate are added, there results a categorical set of axioms for real projective geometry of three dimensions. To obtain complex geometry, Pieri adds, to I–XXII, seven more postulates (XXIII–XXX).   Of these, XXIII is a sort of pseudo-archimedean axiom whose content may be roughly suggested by saying that it postulates that if a, b, c are three distinct points the net of rationality determined by them is everywhere dense on the segment (abc).

At first sight it might seem that Veblen and Young's set of assumptions for complex projective geometry contains fewer postulates and is correspondingly simpler than Pieri's set. However, leaving out of consideration some of the earlier axioms in which Pieri gives such a detailed analysis of certain simple relations of alignment, etc., the question arises whether if Veblen's set seems at first sight to be simpler it is not largely because he makes use of a postulate, assumption C, which leaves much to be desired from perhaps at least two points of view.   There is a curious difference between the way in which assumption C functions for complex space as compared with the way it functions for real space.   Suppose S is a definite real projective space (satisfying A, E and H) whose projective structure is determined in the sense that the question whether or not three points in S lie on the same line has a determinate answer as soon as the three points are themselves determined. If C and R are satisfied in S for one association of points with open cuts as indicated in the statement of assumption C then they are not satisfied for any other such association of points with cuts.   That is to say if, in S, (a) on one line l in one net $R(H_0 H_1 H_\infty)$ there is associated with every open cut (A, B) in $R(H_0 H_1 H_\infty)$ a point P on l such that conditions (1) and (2) of assumption C are satisfied and such that R is also satisfied, and (b) in S, on the same line l in the same net $R(H_0 H_1 H_\infty)$ there is associated with every open cut (A, B) (with respect to the scale H_0, H_1, H_∞) a point $\overline{P}_{(A, B)}$ on l satisfying the same conditions (1), (2) and R, then, for every (A, B), $\overline{P}_{(A, B)}$ is identical with $P_{(A, B)}$.

This uniqueness of choice of the association in question does not however exist for the case of a complex space satisfying A, E, H, C, $\overline{R}$ and I.   Hence it is not determined in advance

just what point must be associated with each cut.   Indeed the reviewer believes that in this case for a given H_0, H_1, H_∞ on a given line l and a given open cut $(\overline{A}, \overline{B})$ on l not related in a certain way to H_0, H_1, and H_∞, one may select, for the $P_{(\overline{A}, \overline{B})}$ to be associated with $(\overline{A}, \overline{B})$ *any point whatsoever* of the line l with the exception of an infinity of points which are (in a certain sense) related algebraically to the points H_0, H_1, H_∞— provided of course that after this selection is made, the points to be assigned to the other open cuts are properly chosen. Thus there appears to be involved here an arbitrary element of a very pronounced character.   If the author claims that in using assumption C for a complex space he does not introduce a new undefined idea, then the reviewer would like to ask the following questions.

Why do the authors of volume I say, on page 1 of that volume, "Since any defined element or relation must be defined in terms of other elements and relations, it is necessary that one or more of the elements *and** one or more of the relations between them remain entirely *undefined*†; otherwise a vicious circle is unavoidable " and why, on page 15 of volume 1 do they say "We consider a class (cf. § 2, page 2) the elements of which we call *points,* and certain undefined *classes of points* which we call lines "?   Why not say merely, "We consider a class the elements of which we call points " and substitute the following in place of the treatment beginning with line 9 of page 16?

"Concerning points we now make the following assumption:

"ASSUMPTION A.   With every two points X and Y there is associated a class of points XY such that

(1)‡ if A, B and C are points which do not belong to XY for any two points X and Y, and D and E $(D \neq E)$ are points such that B, C, D belong to X_1Y_1 for some pair of points X_1 and Y_1 and C, A, E belong to X_2Y_2 for some pair of points X_2Y_2 then there is a point F such that A, B, F belong to some X_3Y_3 and D, E, F belong to some X_4Y_4,

(2)§ If $X \neq Y$ there are at least three points in the class XY.

(3)‖ There do not exist two distinct points X and Y such that every point is in the class XY."

Etc.

Can any objection be made to this procedure which

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* Italics are the reviewer's.
† Italics are the authors'.
‡ Cf. A_3.   § Cf. E_0.   ‖ Cf. E_3.

would not also be an objection to the employment of C as an assumption for complex geometry? If in using such an assumption as C the author does not introduce a new undefined relation, then is it not at least true that in almost (or quite) every case where a set of postulates has ever been constructed in terms of more than one undefined element one can construct a closely corresponding set in terms of only *one* undefined element in a manner similar to that employed in C and in the example that I have given above?

A rather puzzling question now arises. If it be granted that in employing C in the set A, E, H, C, $\bar{R}$, I, the author is really introducing a new undefined relation then is it or is it not true that this is also the case for the set A, E, H, C, R? Or does the mere fact that the above-mentioned arbitrary element is present in the first case and absent in the second—does this fact alone afford sufficient grounds for concluding that the use of C introduces a new undefined relation in the first connection but not in the second?

Aside from the above considerations, assumption C leaves something to be desired from the standpoint of analysis. And what are we to understand is the relation between C, $\bar{R}$ and I? Should $\bar{R}$ be stated as a *separate* assumption? It reads "On some line l not all points belong to the same chain." Here we have a postulate which is stated separately from C but in which there is used a term (chain) which has been defined (?) in terms of an "association" postulated in C. It seems to me the situation would have been made clearer if instead of stating $\bar{R}$ and I separately the author had incorporated in postulate C, after the statement of condition (2), something like the following:

"(3) If in terms of this association of points with cuts 'the chain defined by A, B, C' is defined as indicated on pages 17 and 21 for any three collinear points A, B, C; then there exists a line $\bar{l}$ such that (a) on $\bar{l}$ not all points belong to the same chain, (b) through a point P of any chain C of the line $\bar{l}$ and any point J on $\bar{l}$ but not in C there is not more than one chain on $\bar{l}$ which has no other point than P in common with C."

Of course this strengthened postulate C is more complicated than the original. But does not the attempt to state $\bar{R}$ and I as separate assumptions serve to becloud the true state of affairs?

In an article in the *Annals of Mathematics*, volume 11 (1909), page 34, J. W. Young says "The notion of a chain has been fundamental in the synthetic introduction of imaginaries into geometry since the time of von Staudt. In the more recent work on the foundations of projective geometry it necessarily plays an important rôle. Pieri has indeed recently chosen it as one of the undefined elements in his set of assumptions for complex geometry. More recently Professor Veblen and I have given a set of assumptions for projective geometry in which point and an undefined class of points called a line are the only undefined elements and in which the chain is defined." It is perhaps superfluous to say that the reviewer does not agree with this statement. First there at least seems to be room for debate on the question whether Veblen and Young do not employ a third undefined element. Secondly I certainly do not feel that they have defined chain anywhere in volume I or volume II. If it is true that point and line are the only undefined elements here and that chain is defined, then it would have been possible to have gotten along with point as the only undefined element, both line and chain being defined. One could* even do this on the basis of the single axiom that the cardinal number of the set of all points is C. In this case there would be an arbitrary element involved in the "definitions" of both line and chain. On what grounds if any can one object to this proceedure without objecting to that of Veblen and Young? If we are to allow an arbitrary element in connection with the notion of a chain on what grounds can we object to doing an entirely similar thing in connection with the notion of a line? Perhaps it may be contended that it is undesirable that this sort of arbitrariness should be associated with the notion of a line because this notion is so fundamental in projective geometry. In complex projective geometry the notion of a chain is also quite fundamental.† Indeed I am not sure but that it might be considered just as fundamental as that of a line. It seems to the reviewer that there is much to be said in favor of Pieri's treatment in which point, line and chain are undefined. The question arises however whether, from assumptions C, $\overline{R}$ and I and the results that the authors have established concerning them, one may not perhaps obtain *suggestions* for a well

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* See below.
† Cf., indeed, the above quotation from one of the authors of Volume I.

analyzed set of postulates in terms of point, line and chain or in terms of point, line and order, but simpler than that of Pieri.

On pages 302, 303 and 304 occur three statements which for purposes of reference I will call I, II and III. These statements are as follows:

I. "Assumptions I–IX, XVII are categorical for the euclidean space, i.e., if two sets of objects [P] and [Q] satisfy the conditions laid down for points in the assumptions, there is a one-to-one reciprocal correspondence between [P] and [Q] such that the subsets called lines of [P] correspond to the subsets called lines of [Q]. Thus the internal structure of a euclidean space is fully determined by assumptions I–IX, XVII."

II. "Assumptions I–X, XVII have a different rôle from X–XVI or XVIII–N in that they determine the set of objects (points, lines, etc.) which are presupposed by all the other assumptions. The choice of these assumptions is logically arbitrary. The choice of such sets of "assumptions" as X–XVI is not arbitrary, it must correspond to a properly chosen group of permutations of the objects determined by I–X, XVII."

III. "The point of view of the writer is that if X–XVI or XVIII–N are to be regarded as independent assumptions their independence is of a lower grade than that of I–IX, XVII. They constitute a definition by postulates of a relation (congruence or nearness) among objects (points, lines, etc.) already fully determined."

As to I, what is meant by the "internal structure" of a euclidean space? Do not circles have as much to do with the internal structure of euclidean space as do straight lines? If so it seems to me that the internal structure of a euclidean space is not at all fully determined by assumptions I–X, XVII. What does it mean to say that two sets of objects [P] and [Q] satisfy the conditions laid down for points in the assumptions? The assumptions in question are in terms of point and order (not point alone). In order to give a definite interpretation of a space satisfying I–IX, XVII do we not need to be told just what objects are points for that interpretation and just what points A, B and C are in the order ABC in that interpretation? Grant that we are given such an interpretation S of a space satisfying assumptions I–X, XVII. Then for any three points it is fully determined

whether or not they are on the same line* and to that extent the internal structure of the space S is fully determined. But if one who is curious to know whether the interpretation in question is an interpretation of euclidean geometry should pick out three non-collinear points and ask "are these three points on a circle?"—What answer could be made? One might reply (?) "if you pick out a polar system at infinity I will tell you whether A, B and C are on a circle with respect to that polar system"—but would that be an answer to the question?

As to II and III, if the mere fact that a certain set of postulates is categorical in terms of something, no matter what (I suppose the statement "they determine the set of objects," etc., means that I–X, XVII are categorical in terms of point and order), implies that no further postulates are necessary for any purpose or at least that any further postulates are independent to a lesser degree, if at all, then why not base all of euclidean geometry on the following axiom?

AXIOM A. *The number of points in space is C (the power of the continuum).*

On the basis of Axiom A, straight lines, order and congruence can be so defined that Axioms I–IX of § 29 and Axioms† X–XVI of § 66 will all be fulfilled. Thus Axiom A would be a sufficient basis for three-dimensional euclidean geometry. But straight lines, order and congruence could also be defined in some other way so that all the theorems of two-dimensional euclidean geometry would be fulfilled. With the use of another set of definitions Bolyai-Lobachevskian geometry could be obtained, etc., etc.

With reference to the last two sentences of Statement II, —in what sense, if any, are assumptions I–X, XVII arbitrary? Must not one select II so that it will not contradict I or follow from I, select III so that it will not contradict I or II or follow from them, etc.? Is anything different true of X–XVI? Are they not arbitrary except in that they must be selected so as not to contradict each other or I–X, XVII, and so that no one of them will follow from the others together with I–X, XVII? In a certain sense it may of course be said

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* In § 29 the notion of a line is defined in terms of point and order.

† Cf. "Foundations of Geometry," by Oswald Veblen, in Monographs on Modern Mathematics, edited by J. W. A. Young, New York, 1911. Also R. L. Moore, "Sets of metrical hypotheses for geometry," *Trans. Amer. Math. Society*, vol. 9 (1908), pp. 487–512.

that every time a new (independent) assumption is added there is a decrease in the arbitrariness involved in the selection of an additional assumption (unless one is to have either dependence or a contradiction) and after one has postulated I–X, XVII there is of course less arbitrariness than there was at the beginning. It is true that no assumption can be added to I–X, XVII (without there being a contradiction) if it gives additional information (of a certain type*) concerning point and order alone. But it is allowable to add assumptions that give other information.

On page 71 Veblen narrows a definition of Klein by "assigning to the geometry corresponding to a given group only the theory of those properties which, while invariant under this group, are *not invariant under any other group of projective collineations containing it.*" He adds "This will render the question definite as to whether a given theorem belongs to a given geometry." Unless the reviewer fails to understand the meaning of these statements it appears that according to this test it is not a theorem of euclidean geometry that in a given plane there is only one line parallel to a given line through a point not on that line, nor is it a theorem of euclidean geometry that through two given points there is only one line, and it is not a proposition of double elliptic geometry that every two coplanar lines have two points in common. Apparently the author himself does not always hold to this point of view. For instance on page 70 he says "We have thus considered only very general properties of figures and so have dealt hardly at all with the familiar relations, such as perpendicularity, *parallelism,*† congruence of angles and segments, which make up the bulk of elementary euclidean geometry."

On page 83 in order to secure a complete definition of a planar field of vectors should not the author add to (1) and (2) the stipulations that (3) for each vector V and point A there is only one point B such that V corresponds to the ordered point pair AB and (4) if two ordered point pairs AB and $A'B'$ are not equivalent under the group of translations then their corresponding vectors are distinct? The definition as it stands seems to be satisfied if the number 1 is taken as the vector of *every* point pair.

The present review has been much concerned with a dis-

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* Cf. E. H. Moore, "On the foundation of mathematics," *Science*, vol. 17 (1903), 401–416.

† Italics are the reviewer's.

cussion of certain more or less debatable and delicate questions relating to the foundations of mathematics. Let it be not imagined that these questions bear in any way on the usefulness and interest of the main body of the treatise under review.

In Chapter II the author considers order relations in real projective space. Chapter III is concerned with affine plane geometry, that is to say with the geometry corresponding to the group of all those projective collineations which transform into itself the set of all points not lying on a given line l_∞ of a given projective plane π. One of the most interesting features of the text under review is the way in which various propositions are classified under particular groups. For instance, though of course the affine group doesn ot leave all metrical properties invariant it does leave certain particular ones invariant and in Chapter III the author considers a considerable body of such properties. In particular it is interesting that a theory of equivalence of triangles can be based* on this group.

Chapter IV is concerned with euclidean plane geometry regarded as the geometry corresponding to the group of all those projective collineations that leave invariant a fixed involution (without double points) on a line l_∞ in a real projective plane. Chapter V is concerned with ordinal and metrical properties of conics, Chapter VI with inversion geometry and related topics, including complex chains, Chapter VII with affine and euclidean geometries of three dimensions and Chapter VIII with non-euclidean geometries.

Chapter IX is concerned with theorems on sense and separation, a large body of such theorems being proved on the basis of A, E, S and P without use of C and R. Sense classes of various sorts are defined in terms of elementary transformations. In euclidean space of two dimensions for an ordered set of three non-collinear points an elementary transformation is defined as the operation of replacing one of the three points in question by a point which is joined to it by a segment not meeting the line on the other two. In § 181 it is stated that the notion of right and left-handedness can be extended to curves by a limiting process. For a treatment of sense on curves in two dimensions without the use of such a limiting process reference may be made to an article by J. R. Kline.†

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* Cf. page 96.   Also footnote reference to Wilson and Lewis.
† J. R. Kline, "A definition of sense on closed curves in non-metrical analysis situs," *Annals of Mathematics*, vol. 19 (1918), pp. 185–200.

The reviewer feels that the volume under review is a valuable addition to the as yet rather restricted list of advanced mathematical treatises of high grade published in America.

<div align="right">R. L. MOORE.</div>

NOTES.

AT a special meeting on April 23, 1920, the Council of the American Mathematical Society approved the formation of an American section of the international mathematical union and authorized its committee on the union to take the necessary steps to organize the section. The Council also adopted a resolution that the publication of a journal of mathematical abstracts is very desirable, and authorized its committee on bibliography to take steps toward securing the financial support necessary for such a journal. It was agreed that the representatives of the Society in the division of physical sciences of the National research council should present these projects before the division. Accordingly, at its meetings on April 28–29, 1920, the division adopted resolutions recommending to the National research council that the American section of mathematics organized under the auspices of the division be made the authorized agent of the council in the organization of the proposed international mathematical union, and its representative in that body when organized.

At the same meetings the Mathematical association of America was given the right to nominate one member of the division. The number of members at large was increased by one and Professor G. D. BIRKHOFF was elected as the additional member. Professor OSWALD VEBLEN was elected a member of the executive committee of the division.

The project for the publication of a journal of mathematical abstracts was approved and a committee consisting of Professors L. E. DICKSON, OSWALD VEBLEN, and H. S. WHITE was appointed to work out details and consult with the finance committee of the council as to securing the necessary funds.

A committee was also appointed to secure a revolving fund for the publication of important scientific books and papers commercially unattractive to regular publishing houses. Provision was made for the appointment of research committees