Multiplying (15) by m, we have

$$Cm \cdot n = Ca_{12} = 0.$$

Since $a_{12} \neq 0$, $C = 0$. Likewise multiplying by n we see that $B = 0$. Hence equation (14) becomes

$$A \frac{\partial^2 Z}{\partial u \partial v} = 0.$$

Hence, *the minimum surface is a surface of translation. The necessary and sufficient condition that a surface in hyperspace be a minimum surface is that the minimum lines on it are characteristics.*

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SOME ALGEBRAIC CURVES.

BY DR. JAMES H. WEAVER.

(Read before the American Mathematical Society, April 28, 1917.)

IN the following paper two algebraic curves are set up and some of their singularities are discussed. The author believes them to be new. At least a search through considerable of the literature on curves has failed to reveal them.

I.

Let there be any two distinct points A and B. Let the line joining A and B be drawn, and let the distance $AB = c$. Let there be drawn through A a line l_1 making an angle θ with AB, and let there be drawn through B a line l_2 making an angle $n\theta$ with AB (n an integer). We also consider that AB, l_1, and l_2 are in one plane. Let the intersection of l_1 and l_2 be C. It is required to find the locus of C.

Let A be the origin and let AB be the x-axis. Then the equations of the lines l_1 and l_2 will be

$$(1)\ y = x \tan \theta, \qquad (2)\ y = (x - c) \tan (n\theta)$$

respectively.

After eliminating θ from (1) and (2) we get

$$
(3)\quad \begin{aligned} & x^n - \binom{n}{2} x^{n-2}y^2 + \binom{n}{4} x^{n-4}y^4 - \cdots + (-1)^{n-1/2}xy^{n-1} \\ & = (x-c)\left[\binom{n}{1}x^{n-1} - \binom{n}{3}x^{n-3}y^2 + \cdots + (-1)^{n-1/2}y^{n-1}\right], \end{aligned}
$$

where n is of the form $2k + 1$. A similar form holds if n is of the form $2k$. The theory is the same in either case. (3) is then the equation representing the locus of C. Let us call this curve C_n. If in (2) we replace n by $n - r$, we obtain for (3) a curve of degree $n - r$, which we will call C_{n-r}. ($r = 1, 2, \ldots, n - 1$.)

From (3) it is evident that C_n has an $(n - 1)$-point at the origin. The equations of the $n - 1$ tangents at this point will be given by

$$
(4)\quad \binom{n}{1} x^{n-1} - \binom{n}{3} x^{n-3}y^2 + \cdots + (-1)^{(n-1/2)}y^{n-1} = 0.
$$

The factors of (4) are

$$
y - x \tan (k\pi/n) \quad (k = 1, \cdots, n - 1).
$$

Therefore the tangents to C_n at the point A together with the x-axis divide the angular magnitude about A into $2n$ equal parts.

We will now consider the relation of the fixed point B to the curve C_n. Let us write (3) in homogeneous coordinates. It will then be

$$
(5)\quad \begin{aligned} & x^n - \binom{n}{2} x^{n-2}y^2 + \cdots + (-1)^{n-1/2}xy^{n-1} \\ & = (x - cz)\left[\binom{n}{1}x^{n-1} - \cdots + (-1)^{n-1/2}y^{n-1}\right]. \end{aligned}
$$

The first polar of this curve with respect to the point $B = (c, 0, 1)$ is

$$
(6)\quad \begin{aligned} & x^{n-1} - \binom{n-1}{2} x^{n-3}y^2 - \cdots + (-1)^{n-1/2}y^{n-1} \\ & = (x - cz)\left[\binom{n-1}{1}x^{n-2} - \cdots + (-1)^{n-2/2}y^{n-2}\right]. \end{aligned}
$$

This process may evidently be continued. We may then state the following

Theorem: The rth polar of B with respect to C_n is C_{n-r}.

II.

Again let there be three distinct points A, B, and C on the same straight line l, and through the point C let the line l_1 be drawn perpendicular to l. Let lines l_2 and l_3 be drawn through A and B respectively, and let l_2 and l_3 intersect on l_1. Let l_2 make an angle α with l, and l_3 make an angle β with l, and let a line l_4 be drawn through B, making an angle $n\beta$ with l. Let l_2 and l_4 intersect in D. Then just as in section I, the equation representing the locus of D is

$$(7)\qquad \begin{aligned} &k\left[x^n - \binom{n}{2}x^{n-2}y^2 + \cdots\right] \\ &\qquad = (x-c)\left[\binom{n}{1}x^{n-1} - \binom{n}{3}x^{n-3}y^2 + \cdots\right], \end{aligned}$$

where $k = (a - c)/a$ and $a = AC$, and $c = AB$.

It is then evident that the theorem in section I holds for the curve represented by equation(7).

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ON THE RECTIFIABILITY OF A TWISTED CUBIC.

BY DR. MARY F. CURTIS

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Given the twisted cubic

$$(1)\qquad x_1 = at, \quad x_2 = bt^2, \quad x_3 = ct^3, \quad abc \neq 0;$$

to show that the condition that it is a helix is precisely the condition that it is algebraically rectifiable.

If (1) is a helix, then T/R, the ratio of curvature to torsion, is constant. Denoting differentiation with respect to t by