15. In Professor Veblen's second paper it is proved that there does not exist a deformation on an n-sphere which carries each point of an $(n-1)$-sphere into itself and one of the n-cells bounded by the $(n-1)$-sphere into the other.

F. N. COLE,
Secretary.

THE OCTOBER MEETING OF THE SAN FRANCISCO SECTION.

THE thirtieth regular meeting of the San Francisco Section was held at the University of California, on Saturday, October 27. The attendance included the following seventeen members of the Society:

Professor R. E. Allardice, Dr. B. A. Bernstein, Professor H. F. Blichfeldt, Dr. Thomas Buck, Professor L. E. Dickson, Professor G. C. Edwards, Professor M. W. Haskell, Professor L. M. Hoskins, Dr. Frank Irwin, Professor D. N. Lehmer, Professor W. A. Manning, Professor H. C. Moreno, Professor C. A. Noble, Professor E. W. Ponzer, Professor T. M. Putnam, Dr. Pauline Sperry, Mr. J. S. Taylor.

Professor Manning and Dr. Bernstein were elected chairman and secretary, respectively, for the ensuing year. Professors Lehmer and Blichfeldt and Dr. Bernstein were elected members of the programme committee.

The next meeting of the Section will be held at Stanford University, April 6, 1918. The succeeding fall meeting will be held at the University of California on October 26.

The following papers were presented at this meeting:

(1) Dr. B. A. BERNSTEIN: "On a numero-logical foundation for the theory of probability."

(2) Professor L. E. DICKSON: "Some unsolved problems in the theory of numbers."

(3) Professor L. M. HOSKINS: "The strain of a gravitating, compressible sphere of variable density."

(4) Professor W. A. MANNING: "On the order of primitive groups, IV."

(5) Professor R. M. WINGER: "The rational plane cubic as an application of the theory of involution."

Professor Winger's paper was read by title. Abstracts of the papers follow in the order given above.

1. Dr. Bernstein proposes a set of postulates for the theory of probability based on Boolean logic.

2. With due attention to historical perspective, Professor Dickson discussed various unsolved problems in the theory of numbers, including questions on perfect numbers, amicable numbers of higher order, symmetric functions of the totitives of a composite modulus, Fermat's numbers connected with inscriptible regular polygons, Waring's problem, and the distribution of primes. In particular, the opinion was expressed that various topics in the theory of numbers presented an excellent opening for investigation both by amateurs and professional mathematicians who are not actively engaged upon mathematical investigations. No separate publication of this paper will be made, as the material presented is to be found in the writer's History of the Theory of Numbers now in press by the Carnegie Institution.

3. The problem of the strain of a gravitating sphere by small disturbing forces having a spherical harmonic potential has been completely solved for the case of uniform density, and certain cases of variable density have been solved on the assumption of incompressibility. Professor Hoskins's paper contains the solution for an important class of cases of variable density without restriction of the value of the modulus of compression. The cases covered are those in which the density is a rational integral function of the distance from the center. Reasonable assumptions regarding the density of the earth may be represented to a close approximation by relatively simple functions of this kind. A series of numerical results has been obtained for a case in which the density function is a binomial.

4. In this sequel to former studies in primitive groups it is shown that if a primitive group, not alternating or symmetric, contains a substitution of prime order p and of degree $6p$, $p > 6$, its degree in no case exceeds $6p + 10$. In fact the degree cannot be $6p + 7$ or $6p + 8$ if $p > 7$. If $p = 7$, the limit of the degree is 49. Professor Manning found his task considerably lightened by the following theorem: A simply transitive primitive group in whose subgroup that leaves one letter fixed there is one and only one doubly transitive con-

stituent of degree m must have in this maximal subgroup at least one transitive constituent the degree of which is a divisor ($> m$) of $m(m-1)$. This paper has been offered to the *Transactions* for publication.

5. In this paper Professor Winger shows how the classical properties of the rational cubic, R^3, can be derived quite simply from the theory of involution. The method is then employed in the discovery of new theorems. In particular the contact conics, including the perspective conics, are discussed. The paper closes with some theorems on the hyperosculating curves, i. e., curves whose complete intersections with R^3 fall at a point.

W. A. MANNING,
Secretary of the Section.

ON INTEGRALS RELATED TO AND EXTENSIONS OF THE LEBESGUE INTEGRALS.

BY PROFESSOR T. H. HILDEBRANDT.

(*Continued from page 144.*)

III. STIELTJES INTEGRALS AND THEIR GENERALIZATIONS.

While the Lebesgue integral received almost immediate attention and recognition and found its way rapidly into mathematical literature and thought, it is only recently that the definition of Stieltjes seems to have received the consideration to which it is entitled by virtue of its range of applicability and usefulness. As a matter of fact, in the opinion of the writer, it seems to be destined to play the central rôle in integrational and summational processes in the future.

1. *Definition of the Stieltjes Integral.*—(Cf. Stieltjes (23), pages 71 ff.; Perron (17), page 362; Fréchet (5), pages 45–54; Young (29), pages 131, 137.) A definition for this integral was given first by Stieltjes in his memoir on continued fractions. The integral depends for its value upon two functions $f(x)$ and $v(x)$ defined on an interval (a, b). We suppose that they are both bounded. Then the definition is as follows: