For $l = k$ it follows that

$$a_{k-j,\,j} + ib_{k-j,\,j} = (i)^j(a_{k0} + ib_{k0}) \quad (j = 1, \cdots, k).$$

Then

$$W(z) = \sum_{k=0}^{\infty} (a_{k0} + ib_{k0})(x + iy)^k$$

$$= \sum_{k=0}^{\infty} C_k z^k, \text{ where } (C_k \equiv a_{k0} + ib_{k0}).$$

This completes the theorem.

I am indebted to Professor E. J. Townsend for suggesting the problem.

OHIO UNIVERSITY,
ATHENS, OHIO.

CONCERNING THE COMPLEMENT OF A COUNTABLE INFINITY OF POINT SETS OF A CERTAIN TYPE.

BY DR. J. R. KLINE.

(Read before the American Mathematical Society, December 27, 1916.)

IN his "Grundzüge der Mengenlehre," Hausdorff proved that if E denotes a euclidean space of two or more dimensions while R is a countable set of points belonging to E, then $E-R$ is a connected* point set.† It is the object of the present paper to prove a theorem, which contains Hausdorff's theorem as a special case. Hausdorff's method of proof does not apply for the proof of the more general theorem. While the proof is carried out for the case of two dimensions, it is evident that a similar proof would apply to any higher number of dimensions.

THEOREM. *If M is a domain‡ and G_1, G_2, G_3, $\cdots$ is a countable infinity of nowhere dense§ closed point sets, no one of*

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* A set of points is said to be *connected* if, however it be divided into two mutually exclusive proper subsets, one of them contains a limit point of the other.

† Cf. F. Hausdorff, "Grundzüge der Mengenlehre," Leipzig, Veit, 1914, p. 333.

‡ A domain is a connected set of points M, such that if P is a point of M, then there exists a region containing P and lying in M.

§ A set of points is said to be *nowhere dense*, if in every region R_1 there exists a region R_2 entirely free of points of the set. A set is said to be *closed* if it contains all its limit points.

which disconnects any domain, then $M-(G_1+G_2+G_3+\cdots)$ *is connected.*

*Proof.**—Let A and B denote any two distinct points of $M-(G_1+G_2+\cdots)$.† The point set $M-G_1$ is a domain. For each point P of $M-G_1$, let K_{1_P} denote a region, of subscript greater than or equal to 1, which belongs to the fundamental sequence‡ and is such that K'_{1_P}§ is a subset of $M-G_1$. Call the set of all such regions S_1. There exists a simple chain‖ $R_{11}, R_{12}, R_{13}, \cdots, R_{1n_1}$ from A to B every link of which is a region of the set S_1. Call this chain C_1. As two regions which have a point in common, also have in common a region containing that point, and as G_2 is a nowhere dense closed point set, it follows that R_{1i} and R_{1i+1} $(i=1,2,\cdots,n_1-1)$ have in common a point P_{1i}, not belonging to G_2. Call A, P_{10} and B, P_{1n_1}. For each point P of R_{1i} $(i=1,2,3,\cdots,n_1)$ which does not belong to G_2, let K_{2_P} denote a region of the set $K_2, K_3, \cdots$, which is such that K'_{2_P} lies in R_{1i} and contains no point of G_2. Call the set of all such regions S_2. It is now possible to construct a simple chain C_2, which satisfies all the requirements of Professor Moore's C_2¶ and has the additional property that every link of C_2 belongs to S_2 and therefore contains no points of G_1+G_2. Continue this process. We obtain an infinite sequence of chains $C_1, C_2, C_3, \cdots$ which satisfy all the requirements of Moore's sequence of chains $C_1, C_2, C_3, \cdots$ and have the additional property that no link of the chain C_n has a point in common with the set G_1+G_2

* Our theorem is proved on the basis of the system of axioms Σ_1 proposed by R. L. Moore in his paper, "On the foundations of plane analysis situs," *Transactions Amer. Math. Soc.*, vol. 17 (1916), pp. 131–164.

† That $M-(G_1+G_2+G_3+\cdots)$ contains infinitely many points follows at once from one of the theorems of Baire, *Annali di Mat.* (3), vol. 3, p. 65.

‡ Select once for all a definite sequence, $K_1, K_2, \cdots$ satisfying the conditions of Axiom 1 of Σ_1. This definite sequence will be called the *fundamental sequence* and its regions will be termed *fundamental regions*.

§ The *boundary* of a point set M is the set of all limit points of M, which do not belong to M. If R is a region, R' denotes the point set composed of R plus its boundary.

‖ If A and B are distinct points, then a *simple chain* from A to B is defined by R. L. Moore as a finite sequence of regions $R_1, R_2, R_3, \cdots, R_n$ such that (1) R_i contains A if and only if $i=1$, (2) R_i contains B if and only if $i=n$, (3) if $1\leqq i\leqq n$ and $1\leqq j\leqq n$, while $i<j$, then R_i has a point in common with R_j if and only if $j=i+1$. The region R_k $(1\leqq k\leqq n)$ is said to be the kth *link* of the chain. See R. L. Moore, loc. cit., p. 134. For a proof of the existence of such a chain, see R. L. Moore, loc. cit., Theorem 10, p. 135.

¶ Cf. R. L. Moore, loc. cit., p. 137.

$+ \cdots + G_n$. Let $\overline{C}_n$ denote the point set which is the sum of all the links of the chain C_n, while C denotes the set of all points that the sets $\overline{C}_1, \overline{C}_2, \overline{C}_3, \cdots$ have in common. The point set C is a simple continuous arc* from A to B, lying entirely in the set $M - (G_1 + G_2 + \cdots)$.†

It follows that $M - (G_1 + G_2 + \cdots)$ is connected.

UNIVERSITY OF PENNSYLVANIA,
PHILADELPHIA, PA.

AN ANALOGUE TO PASCAL'S THEOREM.

BY DR. A. L. MILLER.

A DECAGON is said to be doubly inscribed in a cubic if every odd side of the decagon cuts three even sides on the cubic and and every even side cuts three odd sides on the cubic.

That there exist decagons doubly inscribed in a cubic can be seen as follows. Let the decagon D have for sides $s_1, s_2, s_3, \cdots, s_{10}$ and let the cubic be C_3. Let

$$s_1 \text{ meet } s_{10}, s_2, s_4 \text{ on } C_3,$$

$$s_3 \text{ meet } s_2, s_4, s_6 \text{ on } C_3,$$

$$s_5 \text{ meet } s_4, s_6, s_8 \text{ on } C_3,$$

$$s_7 \text{ meet } s_6, s_8, s_{10} \text{ on } C_3,$$

while s_9 is the line joining the third intersection of s_8 with C_3 with the third intersection of s_{10} with C_3. Then, by Cayley's‡ theorem, s_9 also cuts s_2 on C_3.

By a repetition of this last theorem we obtain the following theorem analogous to Pascal's theorem:

If a decagon be doubly inscribed in a cubic the remaining ten intersections of the odd sides with the even ones lie on a conic.

UNIVERSITY OF MICHIGAN.

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* If A and B are distinct points, a *simple continuous arc* from A to B is defined by Lennes as a bounded, closed, connected set of points containing A and B, but containing no proper connected subset containing both A and B. See N. J. Lennes, "Curves in non-metrical analysis situs with an application in the calculus of variations," *American Journal of Mathematics*, vol. 33 (1911) and this BULLETIN, vol. 12 (1906), p. 284.

† For a proof of this statement, see the proof of Theorem 15 of Moore's paper, loc. cit., pp. 136–9.

‡ Cayley: *Cambridge and Dublin Mathematical Journal*, vol. 3.