A SIMPLIFICATION OF THE WHITEHEAD-HUNTINGTON SET OF POSTULATES FOR BOOLEAN ALGEBRAS.

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OF the various sets of postulates that have been given for Boolean logic the most elegant and natural is the set of Huntington's based on Whitehead's "formal laws."* This set may be simplified by reducing the number of its postulates without injuring, the writer feels, the elegance or the naturalness of the original. This reduction is effected by substituting for Huntington's Postulates II_a, II_b, and V the following single postulate:

POSTULATE X. *For any element b in the class there exists an element $\bar{b}$ such that, whatever a is,* $a \oplus (b \odot \bar{b}) = a$ *and* $a \odot (b \oplus \bar{b}) = a$.

Evidently, Huntington's Postulates II_a, II_b, and V follow from Postulate X, with the help of I_a and I_b.

Evidently, also, Postulate X can be derived from II_a, II_b, and V, with the help of I_a, I_b, III_a, and III_b.

It is of course seen that by adopting Postulate X in place of II_a, II_b, and V, not only is the number of Huntington's postulates reduced from ten to eight, but also the number of postulated special elements is reduced from three ("zero," the "whole," and the "negative") to one (the "negative").

In establishing the independence of the modified set of postulates Huntington's systems for I_a, I_b, IV_a, IV_b, VI can serve for the same numbered postulates in the new set. For Postulate X we can take Huntington's system for V. For III_a and III_b, however, a class of more than two elements is, in each case, necessary. Proof-systems for these two postulates are, respectively, the following:

$\overline{III}_a$.

$\oplus$	e_1	e_2	e_3
e_1	e_1	e_1	e_1
e_2	e_1	e_2	e_2
e_3	e_1	e_3	e_3

$\odot$	e_1	e_2	e_3
e_1	e_1	e_2	e_3
e_2	e_2	e_2	e_2
e_3	e_3	e_2	e_3

* See E. V. Huntington, "Sets of independent postulates for the algebra of logic," *Transactions Amer. Math. Society*, vol. 5 (1904), pp. 288–309. The set referred to is the first of the three sets treated by Huntington in his paper.

Here $e_2 \oplus e_3 \neq e_3 \oplus e_2$.

$\overline{\text{III}}_b$.

$\oplus$	e_1	e_2	e_3
e_1	e_1	e_2	e_3
e_2	e_2	e_2	e_2
e_3	e_3	e_2	e_3

$\odot$	e_1	e_2	e_3
e_1	e_1	e_1	e_1
e_2	e_1	e_2	e_2
e_3	e_1	e_3	e_3

Here $e_2 \odot e_3 \neq e_3 \odot e_2$.

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NOTE ON REGULAR TRANSFORMATIONS.

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LET $u(x)$ be bounded and integrable, $0 \leqq x$, and $k(x, y)$ integrable in y for each x, $0 < y \leqq x$; then the transformation*

$$v(x) = \alpha u(x) + \int_0^x k(x, s)u(s)ds \tag{1}$$

is regular if

$$\lim_{x=\infty} u(x)$$

implies the existence of

$$\lim_{x=\infty} v(x)$$

and the equality of the limits. The transformation (1), which depends on the number α and on the function $k(x, y)$, will be denoted by the symbol $[\alpha; k(x, y)]$. Examples of regular transformations are given by [1; 0], which is the identical transformation, and $[0; 1/x]$, which corresponds to the first Hölder mean. In a forthcoming paper† the author discusses conditions on α and $k(x, y)$ for the regularity of the transformation‡ (1), and proves the following theorem:||

THEOREM 1. *A sufficient condition that $k(x, y)$ defined, $0 < y \leqq x$, and integrable in y for each x, correspond to a*

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* It is assumed that the improper integral converges; the lower limit of integration is taken zero for convenience.

† *Transactions*, vol. 17 (1916).

‡ The function $k(x, y)$ in (1) is $(1 - \alpha)$ times the function $k(x, y)$ in the article referred to.

|| See Theorem III in the article referred to; the numbers a and b of that theorem are here replaced by 0 and a respectively. The right-hand member of the last condition is $1 - \alpha$ instead of unity; see preceding footnote.