

Summary.—When the accelerations of three points in a rigid body are given, the acceleration of any point in the plane of the given points is determined uniquely. The acceleration of a point not in the plane of the given points is in general two valued. Moreover, there are in general four sets of values of ω and $\dot{\omega}$ which give the same values for the accelerations of the points in a given plane. For a given value of ω there can be determined the value of the spin for the line joining a given pair of points and hence the relative velocity of the two points can be found.

UNIVERSITY OF MICHIGAN,
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Enquête de "l'Enseignement Mathématique" sur la méthode de travail des mathématiciens. Publié par H. FEHR avec la collaboration de T. FLOURNOY et E. CLAPARÈDE. Deuxième édition conforme à la première suivie d'une *Note sur l'invention mathématique* par H. POINCARÉ. Paris, Gauthier-Villars, et Genève, George, 1912. 8vo. 8+137 pages. Price 5 francs.

Notice sur Henri Poincaré. Par E. LEBON. Paris, Hermann, 1913. 8vo. xlviii pages. Price 2 francs.

Savants du Jour: Henri Poincaré, Biographie, Bibliographie analytique des écrits. Seconde édition entièrement refondue. Par E. LEBON. Paris, Gauthier-Villars, 1912. Royal 8vo. 112 pages. Price 7 francs.

It was in the latter part of 1900 that M. E. Maillet wrote as follows:* "Messieurs les Rédacteurs, Il y aurait, ce me semble, une tentative à faire, pour laquelle *l'Enseignement Mathématique* est à mon avis tout à fait désigné, et dont le succès pourrait rendre de bien grands services aux jeunes mathématiciens. Elle consisterait à ouvrir une sorte d'enquête auprès de savants connus; il s'agirait d'obtenir de chacun d'eux quelques renseignements personnels sur sa méthode de travail et de recherche, ses habitudes, l'hygiène générale qu'il juge la plus propre à faciliter son travail intellectuel, la manière de conduire le plus efficacement ses lectures et d'en tirer le meilleur parti, etc., etc. Je me borne ici à indiquer les grandes

* *L'Enseignement Mathématique*, 1901, tome 3, p. 58.

lignes, tout en reconnaissant que, si mon idée était mise en exécution, il y aurait lieu de pénétrer un peu plus dans le détail.”

As an outcome of the letter of which this is the first paragraph, thirty questions were formulated and published,* and copies were distributed among mathematicians at the international congresses in Heidelberg and St. Louis; a certain number of copies were also addressed to savants of different countries. By 1905 a considerable body of material had been collected with reference to the questionnaire, and during the years 1905–1908† this material was published in suitable synoptic form. The whole was collected and first issued as a pamphlet in 1909.

The results of the pamphlet are deduced from documentary evidence of more than a hundred mathematicians who are, for the most part, living; about a score preferred to have their testimony published anonymously. In the inquiry America is represented by statements of such men as Professors Coolidge, E. W. Davis, Dickson, Emch, F. R. Moulton, Rietz, Snyder, J. W. Young, and Mr. Escott.

Here are some illustrations of the questionnaire and of the responses:

Question 1 (a): At what period of your recollection and under what circumstances did the taste for mathematics take possession of you? To this question 93 replies were received; 35 placed the period before 10 years of age, 43 from 11 to 15 years of age, 11 from 16 to 18 years, 3 from 19 to 20 years, and 1 at 26 years of age. M. Lecat reported, “At $3\frac{1}{2}$ my attention was strongly fixed on the idea of number;” Professor Dickson replied, “At the age of 5. . . . At 12 years of age I had decided to pursue mathematical study.” The well-known facts concerning such infant mathematical prodigies as Clairaut, Gauss, Ampère, and Bertrand are also recalled. While Steiner showed early aptitude for oral calculation and astronomy, it was not till 18 years of age that he even learned to write.

Question 6: Have you sought to learn the genesis of the truths, discovered by you, to which you attach the greatest importance? *Question 7:* To what extent, in your opinion, do chance and inspiration play a part in mathematical discovery? Is this part always as large as it appears? *Question*

* *L'Enseignement Mathématique*, 1902, tome 4, pp. 208–211; tome 6, 1904, pp. 376–378, 481.

† *L'Enseignement Mathématique*. A complete list of references is given in tome 10, 1908, p. 172.

8(a): Have you sometimes remarked that discoveries or solutions in subjects entirely foreign to your researches of the moment have presented themselves to you, and that they corresponded to research formerly fruitless? *Question 8(b)*: Have you ever calculated or solved problems in a dream? Or have solutions and discoveries appeared to you in their completeness on waking up in the morning, when they were vainly sought in the evening or days previously? *Question 9*: Do you estimate that your principal discoveries have been the result of a voluntary endeavor directed in a definite way, or have they occurred to you spontaneously, so to speak?

The replies are summed up as follows (pages 47–48): “Mathematical discoveries—small or great, and whatever their content (new subjects of research, divination of methods or of lines to follow, presentiments of truths and of solutions not yet demonstrated, etc.)—are never born by spontaneous generation. They always presuppose a ground sown with preliminary knowledge and well prepared by work both conscious and subconscious. On the other hand every discovery by its very novelty and originality contrasts forcibly with such a statement and appears all the more surprising when it leaps forth unexpectedly from a very prolonged incubation. We learn, then, that according to the cases and the individuals it is sometimes the unlooked for character, and sometimes the dependence on previous work which strikes the author most when he reflects retrospectively. Whence so many varieties of estimate and equal truth of these two celebrated aphorisms, contradictory in appearance, but expressing the two aspects indissolubly bound together, although of relief often very unequal, of the same process: le génie, c’est l’inspiration; le génie, c’est une longue patience.”

In connection with *Question 8 (b)*, reference might have been made to the composition of Maria Agnesi’s *Instituzioni Analitiche* (1748). “To this difficult task,” H. J. Mozans writes,* “she devoted ten years of arduous and uninterrupted labor. And if we may credit her biographer, she consecrated the nights as well as the days to her herculean undertaking. For frequently, after working in vain on a difficult problem during the day, she was known to bound from her bed during the night while sound asleep, like a somnambulist, make her way through a long suite of rooms to her study, where she wrote

* *Woman in Science*, New York and London, 1913, pp. 144–145.

out the solution of the problem and then returned to bed. The following morning, on returning to her desk, she found to her great surprise, that while asleep she had fully solved the problem which had been the subject of her meditations during the day and of her dreams during the night."

Poincaré's "Note" on Mathematical Discovery is of extraordinary interest. It was delivered, in a conference at the Institut général psychologique in May, 1908, and was first published in the Institut's *Bulletin* for June of the same year. A few months later it was included in the volume entitled *Science et Méthode*.* The opening sentences are as follows:

"The genesis of mathematical discovery is a problem which must inspire the psychologist with the keenest interest. For this is the process in which the human mind seems to borrow least from the exterior world, in which it acts, or appears to act, only by itself and on itself, so that by studying the process of geometric thought we may hope to arrive at what is most essential in the human mind.

"This has long been understood, and a few months ago a review called *l'Enseignement Mathématique* . . . instituted an inquiry into the habits of mind and methods of work of different mathematicians. I had outlined the principal features of this article when the results of the inquiry were published, so that I have hardly been able to make any use of them, and I will content myself with saying that the majority of the evidence confirms my conclusions."

The article deals mainly with just such topics as are suggested by questions quoted above. In the first of its five sections the author considers the following questions and answers: How does it happen that there are people who do not understand mathematics? How is error possible in mathematics? What is mathematical discovery? Poincaré points out that, especially in connection with discovery, intuition of mathematical order is fundamental. "A mathematical demonstration is not a simple juxtaposition of syllogisms; it consists of syllogisms *placed in a certain order*, and the order in which these elements are placed is much more important than the elements themselves. If I have the feeling, the intuition so to speak, of this order so that I can perceive the whole of

* Cf., for example, the English translation by Francis Maitland with a preface by B. Russell, London, 1914, pp. 46-63.

the argument at a glance, I need no longer be afraid of forgetting one of the elements; each of them will place itself naturally in the position prepared for it, without my having to make any effort of memory." "Discovery is discernment, selection," and selection is made by the intuition for order.

"But what I have said up to now," Poincaré remarks in commencing the second section, "is only what can be observed or inferred by reading the works of geometers, provided they are read with some reflection. It is time to penetrate further, and to see what happens in the very soul of the mathematician." And Poincaré recounts his recollections of how he wrote his first treatise on Fuchsian functions. Some of the underlying ideas are developed in the following section. Then there is a summing up in section four. "The result of all that precedes is to show that the unconscious ego or, as it is called, the subliminal ego, plays a most important part in mathematical discovery."

Poincaré considers as a first hypothesis: "The subliminal ego is in no way inferior to the conscious ego; it is not purely automatic; it is capable of discernment; it has tact and lightness of touch; it can select, and it can divine. More than that, it can divine better than the conscious ego, since it succeeds where the latter fails. In a word, is not the subliminal ego superior to the conscious ego?" Poincaré explains why he is loth to give an affirmative answer to this question. He prefers rather to consider: "Of the very large number of combinations which the subliminal ego blindly forms, almost all are without interest and without utility. But, for that very reason, they are without action on the esthetic sensibility; the consciousness will never know them. A few only are harmonious, and consequently at once useful and beautiful, and they will be capable of affecting the geometer's special sensibility I have been speaking of; which, once aroused, will direct our attention on them, and will thus give them the opportunity of becoming conscious." "The conscious ego is strictly limited." Limitations and characteristics of the subliminal ego are considered in the last section.

Such in barest outline are some of the thoughts.

M. Lebon's "Notice" is a reprint in pamphlet form of the introduction to the second edition of Poincaré's *Hypothèses Cosmogoniques*. The first half of the memoir is "Sur la vie

de Henri Poincaré," the latter, " Sur les travaux scientifiques de Henri Poincaré " and here M. Lebon tells us that his aim has only been to " signaler les beaux résultats des recherches originales de Henri Poincaré et les démonstrations rigoureuses qu'il donne, en insistant surtout sur les idées directrices de ses profondes études. D'une part, je souhaite avoir réussi à indiquer nettement comment son esprit fécond et universel est parvenu à élucider les théories obscures, à étendre le domaine des théories naissantes ou à en signaler les défauts. D'autre part, j'espère donner, à ceux qui ont en vue de faire progresser la Science, l'idée de lire avec attention tous ses écrits, qui leur montreront soit la meilleure marche à suivre, soit les points qu'il jugeait dignes d'être encore approfondis."

The first part of the " Notice " is especially interesting. As the topic is less dwelt upon in memorial sketches, a few quotations may be appropriately given in illustration of the whole.

" Henri Poincaré possessed, to a high degree, intuition of mathematical nature. At the Nancy lycée his comrades were struck by it. M. Paul Appell, his codisciple in Mathématiques Spéciales, affirms that he already had ' le don génial d'apercevoir intuitivement, avec le détail particulier de chaque question, l'idée générale dont elle procède, et la place qu'elle occupe dans l'ensemble.' From his first year at the lycée, Henri Poincaré had a method of working all his own. He had to force himself to sit at a study table, and neither noise nor conversation disturbed the working of his mind. To fix his thought on a subject there was no need for auxiliary material; it sufficed that a logical thread pervaded it, in order that it could not escape him. . . .

" Mathematician, philosopher, poet, artist, Henri Poincaré had to be also a great writer. His only aim was to express his thought with all his sincerity and to communicate to his readers his emotions and noblest enthusiasms. He wrote with a dash of the pen, for his ideas were of such a delicate nicety, his thoughts so excessively active, they almost always found immediately their perfect expression. . . .

" The style, infinitely supple and varied, is now the style of the savant, then that of the scholar or of the poet: it is also that of a writer truly French and of the line of the Montaignes, of the Molières, and of the Pascals. Elegant, simple, limpid, of great conciseness, this style abounds in amusing sallies, in an irony

occasionally cutting, but these sallies were aimed at ridiculous things, never at persons. It abounds also in pleasing and picturesque imagery set forth in ordinary language. But Henri Poincaré often happily rejuvenated the common expression by carrying to a conclusion the comparison that it implies or by imbuing a figure employed with originality, freshness, appealing power. . . .

“In scientific matters his only preoccupation was the search for truth. He concerned himself little with glory. He preferred that his name should not be given to any of his discoveries: to contemplate truth for an instant face to face was the only reward which appeared to him worthy of emulation. In acting thus he was certainly obedient to considerations of a lofty order, of an æsthetic order if one may so express it; but he was swayed by an impetuous sense of justice. To him the savants were all soldiers of one army. If in the common contest to ravish nature of her secrets some brilliant captains organize victory, it is nevertheless owing to the discipline, to the courage, to the endurance of the troops, that they win so completely. But, in the combat, how many brave soldiers fall ‘sans laisser de noms, et après avoir utilement aidé à la victoire’”! . . .

“He had the happiness to unite his life to that of an intelligent companion, discreet and devoted, who embellished his existence and facilitated his tasks; for as M. Appell has expressed it, ‘elle entourait son mari de l’atmosphère familiale, profondément unie et calme, qui seule permet les grands travaux de la pensée.’ Henri Poincaré was the most tender and happy of fathers. He saw growing about him the children whom he loved profoundly and who recompensed him by their care always to have lurking about their lips the sweet smile of their affection and their joy. . . .

“As supreme consolation to those who loved him, Henri Poincaré has bequeathed to the centuries to come, with the example of a life as simple as it was beautiful and nobly completed, sa reconfortante pensée, sa foi en la grandeur, en la beauté de l’humanité. Son exemple et son oeuvre ont vaincu le néant.

“‘Recevant d’âge en âge une nouvelle vie,
Ainsi s’en vout à Dieu les gloires d’autrefois;
Ainsi le vaste écho de la voix du génie
Devient du genre humain l’universelle voix. . . .’”

Although the new edition of Lebon's bio-bibliographical work is about a half larger than the old one* its general plan and appearance is the same. To the section on Biography have been added: (1) Darboux's reply to Poincaré's discourse at Darboux's Jubilee; (2) corrections and additions to the list of degrees, honorary titles, decorations, etc.; (3) titles of articles and works on Poincaré. The bibliographical part of Section II, on Mathematical Analysis, has been increased in size by 9 pages through the addition of details concerning new editions, reviews and new titles. In a similar way 5 pages have been added to Section III, on Analytical Mechanics and Celestial Mechanics, 6 pages to Section IV, on Mathematical Physics, and 2 pages to Section V, on Scientific Philosophy. History of Sciences is the title of Section VI, instead of "Necrology" in the old edition; the bibliography has been increased fourfold. Section VII is much the same. The total number of titles is 495, an increase of 59.

This work has been most admirably carried out and is beautifully printed and arranged. Errors and omissions at the date of publication, May 25, 1912, are probably very few. The error (page 85) in the page numbers (583 for 593) of Poincaré's note on non-euclidean geometry in the *Traité de géométrie* of Rouché and Camberousse† still persists. And Russian editions‡ of the conférence on "L'évolution des lois" (page 92) are overlooked.

As practically no reference has been made in this BULLETIN to recent memoirs on Poincaré, I append herewith a list of titles which I have met with, and which supplement Lebon's work. Those memoirs which are signed, are arranged alphabetically according to authors. It will be generally conceded, I believe, that Hadamard's memoir, on Poincaré as a mathematician, is of unsurpassed excellence.

R. d'Adhémar, (1) *Revue des questions scientifiques*, Brussels, vol. 22 (3), 1912, pp. 349-385; (2) Henri Poincaré ("Philosophes et Penseurs" série), Paris, Bloud et Gay, 1914. 12 mo. 64 pp.

* Reviewed by J. W. Young in this BULLETIN, October, 1910, vol. 17 (2), pp. 42-43.

† There are the same page numbers (581-593) for this note in the 8^e éd. of the *Traité*, Paris, 1912.

‡ *Kagan's Bote* (formerly *Spaczinski's Bote*), Nr. 544, pp. 81-89 and Nr. 545, pp. 105-112, 1911. Also separately printed with E. Kohn's "Space and time from the standpoint of physics" under the title, in Russian, "Kohn and Poincaré: Space and time from the standpoint of physics." Odessa, *Mathesis*, 16mo., 81 pp.

—P. Appell, (1) *Revue du mois*, vol. 14, 1912, pp. 129–132; (2) *Annuaire Bureau des longitudes*, 1913, D. 14–18; (3) *Revue scientifique*, vol. 19, 1913, pp. 475–476, vol. 20, 1913, pp. 144–146.—K. Bajev, *Nouvelles de la société russe d'astronomie* (Russian), vol. 7, 1912, pp. 263–269.—R. Berthelot, *Un romantisme utilitaire; étude sur le mouvement pragmatiste*. Tome 1: *Le pragmatisme chez Nietzsche et chez Poincaré*. Paris, Alcan, 1911. (Deuxième partie; un pragmatisme scientifique. le pragmatisme fragmentaire et mitigé de Poincaré, pp. 195–416.)—Bigourdan, *Annuaire Bureau des longitudes*, 1913, D. 19–23.—E. Boutroux, *Revue de Paris*, vol. 20₁, pp. 673–702, vol. 20₂, pp. 77–91, 1913; also reprinted in pamphlet form, Coulomniens, 1913, 47 pp.—P. Boutroux, *Revue du mois*, vol. 15, 1913, pp. 155–183.—H. C. Brown, *Journal of Philosophy, Psychology and Scientific Methods*, New York, vol. 11, 1914, pp. 225–236.—L. Brunschvicg, “Poincaré le philosophe,” *Revue de métaphysique et de morale*, vol. 21, 1913, pp. 585–616 (portrait of Poincaré in 1887).—A. Buhl, *L'Enseignement Mathématique*, vol. 15, 1913, pp. 9–32 (portrait of Poincaré in his study).—R. H. Chassériaud, *Elektrotechnische Zeitschrift*, Berlin, vol. 33, 1912, p. 883.—J. Claretie, *Annuaire Bureau des longitudes*, 1913, D. 3–7.—Cornille, *Annuaire Bureau des longitudes*, 1913, D. 23–25.—S. Dickstein, *Wiadomości matematyczne*, Warsaw, vol. 16, 1912, pp. 249–260 (portrait).—J. Echegazay, *Revista Soc. matem. española*, Madrid, vol. 2, 1912, pp. 33–39 (portrait).—G. Eichhorn, *Jahrbuch der drahtlosen Telegraphie*, Leipzig, 1912, vol. 6, pp. 109–113.—H. Fehr, *L'Enseignement Mathématique*, vol. 14, 1912, pp. 391–392.—M. Fouché, “La philosophie d'Henri Poincaré,” *Bulletin de la Société Astronomique de France*, Paris, vol. 27, 1913, pp. 299–306. (Conférence faite à la séance du 2 Octobre, 1912.)—Galazine, *Bulletin de l'Acad. Impériale des Sciences de St. Pétersbourg* (Russian), 1912, pp. 819–820.—Gust'Haü, *Annuaire Bureau des longitudes*, 1913, D. 1–3.—J. Hadamard, (1) “Poincaré le mathématicien,” *Revue de métaphysique et de morale*, vol. 21, 1913, pp. 617–658 (portrait of Poincaré in 1908); (2) “Henri Poincaré et le problème des trois corps,” *Revue du mois*, vol. 16, 1913, pp. 385–418.—S. C. Haret, *Bulletin* (section scientifique) *Acad. roumaine*, Bucharest, vol. 1, 1912–13, pp. 50–65.—G. Humbert, *La Nature*, vol. 40₂, 1912, pp. 143–144 (1887 portrait, photo by Pirou).—P. E. B. Jourdain, *The Monist*, vol. 22, 1912, pp. 611–615.—A. Korn, *Sitzungsberichte d. mathem. Ges.*, Berlin, vol. 12, 1913, pp.

2-13 (portrait).—E. Lampe, *Naturwissenschaftliche Rundschau*, Braunschweig, vol. 27, 1912, pp. 476-479.—P. Langevin, "Poincaré le physicien," *Revue de métaphysique et de morale*, vol. 21, 1913, pp. 675-718; the same in *Revue du mois*, vol. 16, 1913, pp. 419-463.—A. Lebeuf, "Poincaré l'astronome," *Revue de métaphysique et de morale*, vol. 21, 1913, pp. 659-674.—Lippmann, (1) *Annuaire Bureau des longitudes*, 1913, D. 7-9; (2) *Comptes rendus de l'Acad. d. Sciences*, Paris, vol. 155, 1912, pp. 1280-1283.—A. E. H. Love, *Proceedings of the London Math. Society*, vol. 11 (2), 1913, pp. xli-xlvi.—P. Mansion, *Mathesis*, vol. 2 (4), 1912, pp. 233-238.—L. Marguillan, *Internationale Monatsschrift für Wissenschaft*, vol. 7, 1912, pp. 546-555.—F. Masson, *Revue scientifique*, vol. 18 (5), 1912, pp. 628-629.—C. Meyer, *Anales Soc. scientif. Argentina*, Buenos Ayres, vol. 74, 1912, pp. 125-147.—A. Mieli, *Rivista di filosofia*, vol. 5, 1913, pp. 44-48.—G. A. Miller, *Science*, New York, vol. 36 (2), 1912, pp. 425-429.—K. Mittenzwey, *Nord und Süd*, Berlin, vol. 147, 1913, pp. 53-58.—Morduchaj-Boltovskoj, *Bulletin de l'Université Impériale de Varsovie* (Russian), vol. 24, 1913, pp. 27-80.—L. T. More, "Poincaré and the Philosophy of Science," *The Nation*, New York, vol. 95, 1912, pp. 242-244.—F. R. Moulton, *Popular Astronomy*, vol. 20, 1912, pp. 621-634.—C. Nordmann, *Revue des deux mondes*, vol. 445, 1912, pp. 331-368.—L. Octavio de Toledo, *Revista soc. matem. española*, Madrid, vol. 2, 1912, pp. 26-27.—P. Painlevé, (1) *Revue du mois*, 1912, pp. 132-134; (2) *Annuaire Bureau des longitudes*, 1913, D. 9-13.—E. Pascal, (1) *Giornali di matem.*, vol. 3 (3), 1912, pp. 303-309; (2) *Rendiconto Accad. d. Sc.*, Naples, vol. 18 (3), 1912, pp. 309-313.—E. Picard, *Annales scientif. de l'Ecole normale sup.*, Paris, vol. 30 (3), 1913, pp. 463-482; also printed separately.—G. Rageot, *Les savants et la philosophie*. Paris, Alcan, 1908. (Chapter 2: Le néo-criticisme d'un géomètre Henri Poincaré.)—L. Rougier, *Henri Poincaré et la mort des vérités*. Paris, *La phalange*, 1913, 22 pp.—G. Sarton, (1) *Ciel et terre*, Brussels, 1913, 25 pp. (portrait); (2) *Isis*, vol. 1, 1913, pp. 95-97 (portrait).—J. B. Shaw, "Poincaré as an Investigator," *Popular Science Monthly*, New York, vol. 82, 1913, pp. 209-224.—E. E. Slosson, "Major Prophets of to-day," *Independent*, vol. 71, 1911, pp. 729-741 (portrait); also in volume: *Major Prophets of to-day*, Boston, 1914, pp. 104-146.—W. B. Smith, *The Monist*, vol. 22, 1912, pp. 615-617.—C. Somigliana, *Atti della Reale Accademia della Scienze di*

Torino vol. 49, 1914, pp. 45-54.—J. W. N. Sullivan, *Scientific American*, vol. 107, 1912, p. 78 (portrait).—I. Tschistiakov, *L'Enseignement Mathématique* (Russian), vol. 5, 1912, pp. 197-199.—G. Tzitzéica, *Gazeta matematica*, Bucharest, vol. 17, 1912, pp. 441-445.—O. Veblen, *Proceedings of the American Philosophical Society*, vol. 51, 1912, 9 pp.—V. Volterra, "Henri Poincaré: l'Oeuvre mathématique," *Revue du mois*, vol. 15, 1913, pp. 129-154.*—A. G. Webster, "Poincaré as a mathematical physicist," *Science*, vol. 38 (2), 1913, pp. 901-908.—H. Weyl, *Mathematisch-naturwissenschaftliche Blätter*, vol. 9, 1912, pp. 161-163.†

A few anonymous notes and sketches are to be found in: *Bulletin of the American Mathematical Society*, vol. 19 (2), 1912, p. 43.—*Nature*, vol. 90, 1912, pp. 353-356 (portrait plate supplement and autograph).—*Revue générale des Sciences*, vol. 23, 1912, p. 533.—*Revue scientifique*, vol. 17, 1912, p. 90.—*Sammlung und Mitteilungen und Protokolle der Math. Gesellschaft in Charkow* (Russian), vol. 13 (2), pp. 4-5.—*Supplemento ai Rendiconti del circolo matematico di Palermo*, vol. 8, 1913, pp. 13-32 (reprint of extracts from Lebon's work together with a facsimile of a Poincaré letter).—*Times*, London, July 18, 1912, p. 9, col. c.

In addition to the portraits listed above there is a full-page portrait and autograph in the *American Journal of Mathematics*, vol. 12, 1890; this is the 1887 portrait reproduced in the *Revue de métaphysique et de morale* and *La Nature*. An early portrait is reproduced in *Acta Mathematica*, 1882-1912, *Tables générale des Tomes 1-35*, Upsala et Stockholm, 1913, p. 164. In *Popular Science Monthly*, vol. 82, 1913, p. 412, there is a poor reproduction of the admirably life-like heliogravure frontispiece to Lebon's bio-bibliographical work.

Of the portraits above noted the best are those in the *American Journal etc.*, *L'Enseignement Mathématique*, *Revue de métaphysique etc.*, *Nature*, and Lebon's work.

*The four memoirs by P. Boutroux, Hadamard, Langevin and Volterra, published in *La revue du mois*, have also been issued in book form with five pages of supplementary matter. Paris, Alcan, 1914. 12mo. 2 + 265 pp.

† It may be of interest to add a reference to a sketch of Poincaré prepared by Goursat. This consists mainly of extracts from the writings, mentioned above, of Lebon, Masson, P. Boutroux, V. Volterra, Humbert and Painlevé. The sketch is to be found in *La vie et les travaux des savants modernes par A. Rebière*; troisième édition revue et augmentée par E. Goursat. Paris [1913], pp. 188-203.

It is to be hoped that in the next edition of this work M. Lebon may be moved to give a list of papers and books which have been inspired by Henri Poincaré's suggestions and discoveries.

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SHORTER NOTICES.

First-Year Mathematics for Secondary Schools. By ERNST R. BRESLICH. Chicago, The University of Chicago Press. 1915. 344 pp.

ABOUT six hundred fifty years ago Roger Bacon gave voice to his feelings with respect to the teaching of mathematics, and this voice was in no respect uncertain nor was it at all lacking in emphasis. His words may be found in the *Opus Majus*, in the *Opus Tertium*, and in the manuscripts as yet unpublished of his *De Communia Mathematicæ*. In the last-mentioned work Bacon says that students are burdened with unnecessary difficulties to such a degree that they come to despise mathematics, whereas, if properly taught, the subject could be understood without any unreasonable expenditure of time; and that the first course in mathematics should not be designated as geometry, arithmetic, and so on, but as the elements of mathematics, a preliminary to the special branches.

What Bacon had to say on this phase of teaching was not new; others had said it before, and thousands have said it since, and after a fashion many have put the idea into practice. And so the effort of Mr. Breslich comes to the teaching profession as merely an ancient one clad in new guise. This does not in the least detract from the laudable nature of the effort, but it serves to give the work a kind of historical setting which assists us in judging of its novelty and its probable effect upon education.

The central idea of the work seems to be to select those features of secondary mathematics which are easily within the reach of beginners, postponing the consideration of the more difficult ones to a later period. As the author puts it, "The simpler principles are best suited for beginners, and may therefore be brought together in an introductory course."