THE EIGHTH REGULAR MEETING OF THE SOUTHWESTERN SECTION.

THE eighth regular meeting of the Southwestern Section of the Society was held at the University of Nebraska, Lincoln, Nebraska, on Saturday, November 28, 1914. About twenty-five persons attended the sessions, including the following sixteen members of the Society:

Professor Henry Blumberg, Professor W. C. Brenke, Professor E. W. Davis, Professor H. C. Feemster, Professor A. B. Frizell, Dr. Elizabeth B. Grennan, Professor E. R. Hedrick, Professor Louis Ingold, Professor O. D. Kellogg, Dr. S. Lefschetz, Professor U. G. Mitchell, Professor W. H. Roever, Professor Oscar Schmiedel, Professor E. B. Stouffer, Professor C. E. Stromquist, Professor W. D. A. Westfall.

The morning session opened at 10 A.M. and the afternoon session at 2 P.M. Professor Davis presided. It was decided to hold the next meeting of the Section at Washington University in St. Louis on Saturday, November 27, 1915. The following program committee was appointed: Professor W. H. Roever (chairman), Professor J. N. Van der Vries, Professor O. D. Kellogg (secretary). Attending members were given a smoker on Friday evening at the Commercial Club, and a lunch at the same place on Saturday.

The following papers were presented at this meeting:

(1) Professor P. J. DANIELL: "The end correction for an open cylinder."

(2) Professor OSCAR SCHMIEDEL: "Multiplication and division by variable operators."

(3) Professor LOUIS INGOLD: "Note on surfaces with a single first normal."

(4) Dr. S. LEFSCHETZ: "Double integrals of Picard for an algebraic variety."

(5) Dr. S. LEFSCHETZ: "The equation of Picard-Fuchs for an algebraic surface with arbitrary singularities."

(6) Professor HENRY BLUMBERG: "On the factorization of certain polynomials and certain linear homogeneous differential expressions."

(7) Professor A. B. FRIZELL: "The well-ordering of infinite permutations."

(8) Dr. H. M. SHEFFER: "Deductive systems and postulate theory; I. finite case" (preliminary communication).

(9) Professor E. R. HEDRICK and Miss E. A. WEEKS: "On a definition of discrete oscillation."

(10) Professor W. C. BRENKE: "Convergence of an infinite determinant."

(11) Professor E. W. DAVIS: "The distance between two complex points in ordinary space."

In the absence of the authors, Professor Daniell's paper was presented by Professor Kellogg, and Dr. Sheffer's by Professor Hedrick. Abstracts of the papers follow.

1. When a current flows out of a cylindrical tube of radius a into a large bath, a certain correction ka, where k is some fraction, has to be added to the length of the tube in calculating the resistance. Rayleigh in his Theory of Sound has shown that $.785 < k < .849$ and, in the appendix, that $k < .8242$. Certain integrals containing Bessel functions are evaluated by means of asymptotic series, and then the approximate solution of an infinite number of linear equations in an infinite number of variables, obtained from the usual minimum condition, yields the value of k to any required accuracy. Professor Daniell has found by this means that $.8225 < k < .8232$.

2. In this paper, Professor Schmiedel considers a method of multiplication and division by variable multipliers and divisors for the purpose of extending certain tables which are functions of two arguments to negative integral values of both arguments, and makes application of the method to classes of tables of which the ordinary binomial coefficients are a special case.

3. In a previous paper Professor Ingold has shown that for the proof of the formulas of Gauss and Codazzi the usual requirement, that the n-dimensional space under consideration lie in a linear space of $n + 1$ dimensions, can be replaced by the assumption that the given space have a *single first normal* (i. e., a normal vector which is linearly expressible in terms of the tangent vectors and their *first* derivatives). No example, however, was given of a locus satisfying the last requirement without satisfying the first. In this note such examples are furnished for n-dimensional loci, and it is shown that the

only two-dimensional loci of this character are surfaces generated by the tangents to a curve in space of four or more dimensions.

4. In this paper Dr. Lefschetz deals with integrals of the type $\int\int [A(x, y, z, t)dydz + Bdzdx + Cdxdy]$ for which

$$\frac{\partial A}{\partial x} + \frac{\partial B}{\partial y} + \frac{\partial C}{\partial z} = 0.$$

A, B, C are rational on the variety V having for equation $f(x, y, z, t) = 0$. It is shown that, as could be expected, the number of integrals of this type which are of the second kind proper is $R_2 - \rho$, R_2 being the two-dimensional connectivity of V, ρ the number of its simple integrals of the third kind. The following interesting result is obtained: if A, B, C are of the type

$$\frac{\partial R}{\partial y} + \frac{\partial R'}{\partial z}, \quad \frac{\partial S}{\partial z} + \frac{\partial S'}{\partial x}, \quad \frac{\partial T}{\partial x} + \frac{\partial T'}{\partial y},$$

then there exist three *rational functions* of (x, y, z, t), say α, β, γ such that

$$A = \frac{\partial \beta}{\partial z} - \frac{\partial \gamma}{\partial y}, \quad B = \frac{\partial \gamma}{\partial x} - \frac{\partial \alpha}{\partial z}, \quad C = \frac{\partial \alpha}{\partial y} - \frac{\partial \beta}{\partial x}.$$

5. Dr. Lefschetz's second paper appears in full in this number of the BULLETIN.

6. Let

$$P = p_0(x)\frac{d^ny}{dx^n} + p_1(x)\frac{d^{n-1}y}{dx^{n-1}} + \cdots + p_n(x)y$$

be a linear homogeneous differential expression belonging to the domain $\mathfrak{R}$ of rational functions (i. e., whose coefficients are elements of $\mathfrak{R}$). Denote by d_0 $d_1, \cdots, d_n$ the degrees of $p_0(x)$, $p_1(x)$, $\cdots$, $p_n(x)$ respectively. Professor Blumberg proves the following theorem:

If $d_n - d_0 > 0$ and $d_\nu - d_0 \leqq 0$ $(\nu \neq n)$, P is not expressible as a symbolic product RS, where R and S are linear homogeneous differential expressions belonging to $\mathfrak{R}$ and of the sth and rth order respectively, unless $d_n - d_0 \equiv 0 \text{ mod. } (n/[r, s])$.

Here $[r, s]$ = G.C.D. of r and s. As a particular case we have the theorem

If $d_n - d_0 > 0$, $d_\nu - d_0 \leqq 0$ $(\nu \neq n)$ and $[d_n - d_0, n] = 1$, P is irreducible.

The proofs are algebraic in character, no integrals being used. Moreover corresponding theorems hold for the factorization of the polynomial P in y

$$P = p_0(x_1, x_2, \cdots, x_k)y^n + \cdots + p_n(x_1, x_2, \cdots, x_k),$$

where the coefficients are rational functions of their arguments. The application of the results to the integrals of the corresponding differential equations and to the algebraic functions defined by $P = 0$ is evident. Generalizations are given.

7. By an extension of the process developed in a former paper (Chicago, April, 1914) Professor Frizell shows that the possibility of well-ordering the permutations of an ω-series would carry with it the existence of an ω-series of ordinal types—of which the type of this set of permutations is the lowest—whereof no one can be put into one-to-one correspondence with its predecessor.

8. By means of various "transformations of relational coordinates," and corresponding definitions of equivalence of systems, Dr. Sheffer develops a general postulate theory for deductive systems involving a finite number of elements.

9. In generalizations of the theory of functions of real variables, it seems desirable to suppress as far as possible the notion of length. Such other concepts as that of oscillation are therefore difficult of extension. In this paper, Professor Hedrick and Miss Weeks define a concept which will be called discrete oscillation, and they show that many of the usual theorems concerning oscillation remain true for functions of a real variable when this new concept is substituted for the ordinary oscillation. For functions of a real variable the discrete oscillation is defined in terms of any monoton decreasing set of numbers $a_1, a_2, a_3, \cdots, a_n, \cdots$ which approach zero, and is by definition equal to a_i if the ordinary oscillation ω satisfies the inequality $a_{i-1} > \omega \geqq a_i$. Among theorems whose validity is proven are the theorem that the points at

which the oscillation is not less than a given positive number form a closed set; the Du Bois-Reymond theorems on integration; the Sierpinski theorem; and others. The new concept lends itself readily to broad generalizations, and its simplicity suggests the possibility of advantageous use even in the usual theory.

10. Let a_{ik} be the general element of the infinite determinant D and assume the convergence of $\Sigma|a_{ik}|$. By comparison with an infinite product Professor Brenke obtains the following results, of which (*d*) is a well-known theorem, from which also (*a*) might be derived: (*a*) D converges absolutely to the value 0; (*b*) if the elements of any number of rows or columns of D are replaced by quantities less in absolute value than a positive constant, the new determinant converges absolutely to 0; (*c*) if all the elements a_{ik}, $i > k$, are replaced by quantities less in absolute value than 1, the new determinant converges absolutely; (*d*) von Koch's "normal determinant" converges absolutely; (*e*) a normal determinant remains absolutely convergent if elements a_{ik} are replaced as in (*c*).

11. Professor Davis shows that if the difference between two complex vectors in space is $\delta_1 + \sqrt{-1}\delta_2$ and if k is $UV\delta_1\delta_2$, then the square of the distance between the complex vectors is $e^{\sqrt{-1}\theta}T(\delta_1 + k\delta_2)T(\delta_1 - k\delta_2)$ where θ is the angle between $\delta_1 + k\delta_2$ and $\delta_1 - k\delta_2$. This is an extension to space of a formula of Laguerre.

O. D. KELLOGG,
Secretary of the Section.

NOTE ON THE POTENTIAL AND THE ANTI-POTENTIAL GROUP OF A GIVEN GROUP.

BY PROFESSOR G. A. MILLER.

(Read before the American Mathematical Society at Chicago, December 29, 1914.)

§ 1. *Introduction.*

WITH every regular substitution group there may be associated a conjugate substitution group on the same letters