

tional field which passes through  $P_0$ . (See Kasner's Princeton Colloquium Lectures, page 9, second footnote.)

By a well-known theorem the curvature of a line of force of the weight field is the logarithmic derivative of  $g$  in the direction on a level surface in which  $g$  increases most rapidly, i. e.,

$$\frac{\partial}{\partial x} \log g = \frac{\partial g / \partial x}{g}.$$

From the above relations

$$(3) \quad \frac{1}{2\rho_a} = \left( \frac{\partial g}{\partial x} \right)_0 / g_0, \quad \frac{1}{2\rho_{c''}} = \frac{1}{6g_0} \left[ \left( \frac{\partial g}{\partial x} \right)_0 - 4\omega^2 \sin \phi_0 \cos \phi_0 \right],$$

and hence by (2) we get (1).

For the data:  $h = 49,024$  cm.,  $\phi_0 = 45^\circ$ , and for the potential function for which the Bessel ellipsoid is a level surface and the formula of Helmert gives the acceleration,

$$g_0 = 980.6, \quad \left( \frac{\partial g}{\partial x} \right)_0' = 8.1568 \times 10^{-9}, \quad \omega^2 = 5.3173 \times 10^{-9},$$

and hence by formula (1)

$$S. D. = + .021 \text{ cm.}$$

F. N. COLE,  
*Secretary.*

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#### THE TWENTY-FOURTH REGULAR MEETING OF THE SAN FRANCISCO SECTION.

THE twenty-fourth regular meeting of the San Francisco Section of the Society was held at Stanford University on October 25, 1913. Twenty-one persons were present, including the following members of the Society:

Professor R. E. Allardice, Mr. B. A. Bernstein, Professor H. F. Blichfeldt, Dr. Thomas Buck, Professor G. C. Edwards, Professor G. I. Gavett, Professor Charles Haseman, Professor M. W. Haskell, Professor L. M. Hoskins, Dr. Frank Irwin, Professor D. N. Lehmer, Professor W. A. Manning, Professor H. C. Moreno, Professor C. A. Noble, and Professor E. W. Ponzer.

Professor Edwards, chairman of the section, presided at the opening of the morning session, Professor Manning, chairman-elect, then took the chair. The following officers of the Section were elected for the ensuing year: chairman, Professor Manning, secretary, Dr. Buck; program committee, Professors Blichfeldt, Lehmer, and Dr. Buck.

It was voted to hold the spring meeting at Seattle provided the Pacific Coast Association of Scientific Societies meets there, and the fall meeting at the University of California on October 24, 1914. A committee, consisting of Professors Haskell, Allardice, Blichfeldt, Lehmer, and Noble, was appointed to make arrangements on the part of the Section for the summer meeting of the Society at San Francisco in 1915.

The members present lunched together between sessions.

The following papers were presented at this meeting:

(1) Professor L. M. HOSKINS: "Note on the motion of a freely falling body."

(2) Professor M. W. HASKELL: "Second note on the Del Pezzo quintic."

(3) Mr. B. A. BERNSTEIN: "A complete set of postulates for the logic of classes expressed in terms of the operation 'exception,' and a proof of the independence of a set of postulates due to Del Ré."

(4) Professor M. W. HASKELL: "On the singularities of twisted curves."

(5) Professor W. C. EELLS: "Number systems of the North American Indians."

(6) Professor G. A. MILLER: "Some properties of the group of isomorphisms of an abelian group."

(7) Professor W. A. MANNING: "On the class of doubly transitive groups."

In the absence of the authors, the papers of Professors Eells and Miller were read by title. Abstracts of the papers follow below.

1. One of the simplest methods of studying the motion of a freely falling body is to refer the motion to axes fixed in the earth, making use of the relation between the accelerations of a particle referred to fixed and moving axes. The equations for the motion relative to the earth may thus be formed by introducing, in addition to the actual forces, the fictitious "centrifugal" and "compound centrifugal" forces. The

former may be taken account of by adding to the earth's attraction potential  $V$  a potential  $U = \frac{1}{2}\omega^2\rho^2$ ,  $\omega$  denoting the earth's angular velocity, and  $\rho$  distance from the axis of rotation. For a body falling from rest the effect of the compound centrifugal force is easily computed to a close approximation, the main effect being an easterly displacement proportional to the cube of the time of falling; while a very minute secondary effect would be a deviation toward the equator, proportional to the fourth power of the time. These displacements are superposed upon those due to the field of force corresponding to the potential  $V + U$ . The question whether this field would cause a deviation northward or southward from a plumb-line suspended from the point where the body starts, is closely connected with the question of the curvature of the lines of force in this field; the path would lie on the convex side of that line of force passing through the position of rest, while the plumb-line would hang on the concave side. So far as the effect of this field is concerned, the body will fall on that side of the plumb-line toward which the lines of force are convex. Analysis indicates that the convexity is toward the equator.

A matter of interest in the rigorous analysis is the fact that the motion in the meridian plane is identical with the plane motion of a particle in a stationary field, of force of potential  $V - U\rho_0^4/\rho^4$ ,  $\rho_0$  being the initial value of  $\rho$ .

The analysis is easily extended to the case in which the body has an initial velocity.

2. This paper of Professor Haskell contains the complete form of the Hessian of the given quintic and its reduction to a form which shows that the 5 cusps and the 5 inflexions of the quintic are the complete intersection of the quintic with a certain covariant cubic.

3. Mr. Bernstein presents a set of six postulates all expressed in terms of the single undefined operation "exception,"—the operation which forms from the two logical elements  $a, b$  the element ordinarily denoted by  $a\bar{b}$  and read, " $a$  except  $b$ " or " $a$  which is *not*  $=b$ ." The postulates are proved to be consistent, independent of one another, and "sufficient" for the logic of classes.

The writer also proves the independence of a set of postulates for the algebra of logic due to Professor Del Ré.

4. In this paper Professor Haskell considers the generalization of a theorem due to Clebsch on the singularities of plane curves when given in parameter form, and gives simple criteria for the existence and enumeration of the various singularities of twisted curves.

5. Professor Eells' paper is based upon a study of the number systems of over 300 Indian languages of North America. The evidence for and against a digital origin of counting is presented; the use of additive, subtractive, multiplicative, divisive, and duplicative principles in the formation of number systems is analyzed; examples of decimal, vigesimal, quinary, octonary, quaternary, and ternary systems of numeration are given and discussed; and other noteworthy features of primitive number systems are mentioned. The paper will appear serially in the *American Mathematical Monthly*.

6. As the group of isomorphisms of any abelian group is the direct product of the groups of isomorphisms of its Sylow subgroups, Professor Miller confined his attention to a consideration of the group of isomorphisms  $I$  of an abelian group  $G$  of order  $p^m$ ,  $p$  being a prime number. After determining the number of the Sylow subgroups of order  $p^m$  contained in  $I$ , it was proved that each of these Sylow subgroups is transformed into itself under  $I$  by a group of order  $p^m(p-1)^\lambda$ , where  $\lambda$  is equal to the total number of the invariants of  $G$ . This group of order  $p^m(p-1)^\lambda$  contains an abelian subgroup of order  $(p-1)^\lambda$ , which is the direct product of  $\lambda$  cyclic subgroups of order  $p-1$ .

A necessary and sufficient condition that  $I$  contains only one subgroup of order  $p^m$  is that no two of the invariants of  $G$  are equal to each other. Whenever  $G$  contains at least two equal largest invariants then  $I$  contains an operator whose order is equal to these invariants. If the number of each set of equal invariants of  $G$  is less than  $p$ , then the group of isomorphisms of  $G$  cannot involve any operator whose order is a power of  $p$  and exceeds the largest invariant of  $G$ . When the Sylow subgroups of order  $p^m$  are abelian, either  $G$  must be cyclic or  $m$  must be less than 3.

7. If  $n$  is the degree and  $u$  the class of a doubly transitive group, Bochert's theorem asserts that  $u$  is greater than  $\frac{1}{3}n$ —

$\frac{2}{3}\sqrt{n}$ . This is based upon the fact that two non-commutative substitutions of degree  $u$  cannot have less than  $\frac{1}{3}u$  letters in common. Professor Manning called attention to the case in which at least one of the substitutions of degree  $u$  in the group is of order 2. Here two non-commutative substitutions of degree  $u$  have at least  $\frac{1}{2}u$  letters in common, from which he concludes that  $u$  is greater than  $\frac{1}{2}(n - \sqrt{n}) - 1$ .

THOMAS BUCK,  
*Secretary of the Section.*

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### THE SEVENTH REGULAR MEETING OF THE SOUTHWESTERN SECTION.

THE seventh regular meeting of the Southwestern Section of the Society was held at the University of Missouri, Columbia, Mo., on Saturday, November 29, 1913. About twenty-five persons attended the meeting, including the following sixteen members of the Society:

Professor L. D. Ames, Professor C. H. Ashton, Dr. Henry Blumberg, Professor W. C. Brenke, Professor E. W. Davis, Dr. E. L. Dodd, Dr. Otto Dunkel, Professor E. R. Hedrick, Professor Louis Ingold, Professor O. D. Kellogg, Dr. A. J. Kempner, Professor W. H. Roever, Professor H. E. Slaught, Professor J. N. Van der Vries, Miss Eula Weeks, Professor W. D. A. Westfall.

The morning session opened at 10.30 A.M. and the afternoon session at 2 P.M. Professors Hedrick and Slaught presided. It was decided to hold the next meeting of the Section at the University of Nebraska on November 28, 1914. The following programme committee was elected: Professor E. W. Davis (chairman), Dr. S. Lefschetz, Professor O. D. Kellogg (secretary). Those present attended a smoker at the house of Professor Kellogg on the evening before the meeting.

The following papers were presented:

- (1) Professor W. C. BRENKE: "An example of Abel's integral equation with discontinuous solution."
- (2) Professors E. R. HEDRICK and LOUIS INGOLD: "Generalization of Taylor's series."
- (3) Dr. S. LEFSCHETZ: "Double integrals of the third kind attached to an algebraic variety."