mentioned only in a brief introduction of nine pages, where the solution of certain problems in oscillation is reduced to expansions in terms of normal functions (Eigenfunktionen) belonging to integral equations. The entire text is then taken up with the development of the theory of the solution of linear integral equations following the method of successive approximations. This method may, however, without too much strain on the sense of the terms, be called the method of development in series of oscillating functions, and possibly this idea may have suggested the title.

The treatment is divided into three sections, presenting respectively the theory of linear integral equations with continuous, symmetric kernel, with discontinuous symmetric kernel, and Fredholm's solution of linear integral equations with any continuous kernel. In an appendix some generalizations are given for kernels with discontinuities for many-dimensional problems, and for systems of linear integral equations. The book closes with a bibliography of memoirs by the author relating to the applications of the method of successive approximations. This method owes its development to C. Neumann, Poincaré, Picard, and Korn.*

In a sense the book is an introduction to the theory of developments in series of normal functions particularly when the kernel is unsymmetric. These are, up to the present time, special investigations which each extend the field a little. In many cases of particular forms of the kernel, it is possible to develop theorems analogous to those on which is based the solution of the case of a symmetric kernel. How far this method may lead one in such investigations remains for the future to show.

JAMES BYRNIE SHAW.

Etude sur l'Assurance complémentaire de l'Assurance sur la Vie. By P. J. RICHARD. Paris, A. Hermann et Fils, 1911. 118 pages.

THE appearance recently in this BULLETIN of the note that "at the University of Göttingen . . . candidates in applied mathematics must henceforth be prepared to be examined in the mathematics of insurance," the approval by the Italian Chamber of Deputies of the bill providing for a state monopoly

* H. Bateman, Report on the history and present state of the theory of integral equations, page 21.

of life insurance, and the recent British National Insurance Act, make more generally interesting the monograph before us.

The question is raised as to whether the insurance companies are going to be viewed sometime much as the express companies are to-day. When the ratio of management expenses (exclusive of taxes) to premium income for our American companies varies from 8.7 to 28.4, even to 227.6 for a state company, with an average of 22.4; when the company for which this ratio is below 20 is thought to be quite economically managed, one wonders whether the cost of insurance is not much too high. If the government had the monopoly of life insurance, as in some countries, would it not benefit our people as a parcels post does other people? If the company that has cut its cost of management to half of that of most of the best of other companies could flourish since 1759, and if the cost in Germany is still less, why should not our millions of people profit by what national control could save? The similarities between express and insurance companies cannot be considered here, neither can the causes of the great variation in cost of management.

The author of the study, in an introduction of six pages, states the object and aim of life insurance, calls attention to the possible failure to maintain it, and asks the question which has embarrassed many an agent: "If I fall ill and am no longer able to earn anything, who will pay my premiums?" This leads to the matter in hand, l'assurance complémentaire, which provides against the lapsing of a policy on account of temporary or permanent incapacity for labor due to accident or to disease. The premiums are guaranteed and the amount thus paid beforehand is deducted from the amount paid to the benefactor at the time of death of the insured. The history of this sort of insurance is briefly told. Reference is made to the fact that German and American companies sell such a contract. The regular life companies of France are warned by the Controle Central des Compagnies au Ministère du Travail not to enter into such contracts. The author states that the aim of the study is to show that this risk is as rightfully assurable as the risk of death. The companies have made a serious and rational study only as to equitable charges for the new combination. Though definite statistical data are lacking, the data furnished by other branches of insurance may be temporarily used to outline a rudimentary theory.

At present the companies are simply feeling their way to a reasonable tariff.

I have found about four American companies that have some form of select life policy, or clause in a regular policy, covering this field. It may be of considerable interest to many readers to see a sample clause of this sort:

"*Waiver of Premiums.*—The company, by endorsement hereon, will waive payment of the premiums thereafter becoming due, if the insured, before attaining the age of sixty years and after paying at least one full annual premium and before default in the payment of any subsequent premium, shall furnish proof satisfactory to the company that he has become wholly and permanently disabled by bodily injury or by disease so that he is and will be permanently, continuously and wholly prevented thereby from performing any work for compensation or profit, or from following any gainful occupation. Any premiums so waived shall not be deducted from the sum payable under the policy. Provided that, notwithstanding proof of disability may have been accepted by the company as satisfactory, the insured shall at any time, on demand, furnish the company satisfactory proof of the continuance of such disability; and if the insured shall fail to furnish such proof, or if it shall appear to the company that the insured is able to perform any work or to follow an occupation whatsoever for compensation, gain or profit, all premiums thereafter falling due must be paid in conformity with this contract.

"Without prejudice to any other cause of disability, the entire and irrecoverable loss of the sight of both eyes, or the severance of both hands above the wrists, or both feet above the ankles, or of one entire hand and one entire foot will be considered as total and permanent disability within the meaning of this provision."

The clear, concise statement characteristic of the whole treatment does much toward holding the reader's interest through the five chapters of the book, as the author treats minutely of the conditions of the complementary insurance and even as he calculates tables of morbidity, of healthy and invalid persons, of value of an indemnity for one and for two insured, of premiums for numerous particular forms of insurance, etc. Though with insufficient statistics the author seems to have presented a rational study of this interesting and very

little known form of insurance, pointing out, it would seem, every exigency that could possibly arise.

CHARLES C. GROVE.

Konstructionen und Approximationen. Von THEODOR VAHLEN. Teubner, Leipzig und Berlin, 1911. xii + 349 pp.

ONE who expects to find in this book—Band XXXIII of the Teubner Sammlung—a more or less complete list of constructions and approximations with a strong flavor of applied mathematics will be disappointed, as was the reviewer. According to the author it is intended to help bridge the gap which exists between the mathematics of the German gymnasium and university. The latter does not begin its work where the former ends. Various books, notably those by Klein and Enriques, have been published recently which might be studied by those intending to follow the lectures on higher mathematics. The book under review aims to furnish such preparation by having the student actually come in contact with some concrete facts in mathematics and to know these so well that later when the professor during his lecture has him soaring more or less he may still have a point or two of contact with the earth below.

The class of books having in view preparation for the university is decidedly different for Germany than for the United States. To illustrate this we might mention that the first 75 pages of the book under review are devoted to having the student obtain definite notions concerning the fundamental principles of projective geometry. Special emphasis is placed in all of its phases, both algebraic and geometric, on the interpretation of the cross-ratio. Good drill work, all of it. Of course, it couldn't be included in the lectures given later—that would seem too much like teaching.

Throughout the book the various aspects of the solutions of the three famous problems of antiquity are presented and many references to the literature on the subject given. Interesting metric cubic constructions in which algebra and geometry are closely correlated are cited. Approximate solutions of cubics and biquadratics are obtained geometrically and the limited range of constructions possible with ruler and compass pointed out. In this connection are included several solutions, ancient and modern, of the duplication of the cube and tri-