be convergent and

$$\lim_n \sum_p | x_{np} |^m = \sum_p | x_p |^m$$

is that

$$\lim_n \sum_p | x_{np} - x_p |^m = 0.$$

In the proof of this theorem there arises another necessary and sufficient condition concerning itself with the limit of the sum of a sequence of series of positive terms.

H. E. SLAUGHT,
Secretary of the Section.

THE TWENTY-FIRST REGULAR MEETING OF THE SAN FRANCISCO SECTION.

THE twenty-first regular meeting of the San Francisco Section of the Society was held at Stanford University, on Saturday, April 6, 1912. About fifteen persons were present, including the following members of the Society:

Professor R. E. Allardice, Mr. B. A. Bernstein, Dr. Thomas Buck, Professor H. F. Blichfeldt, Professor G. C. Edwards, Professor M. W. Haskell, Professor L. M. Hoskins, Professor D. N. Lehmer, Professor H. C. Moreno, Professor E. W. Ponzer.

Morning and afternoon sessions were held, Professor Hoskins, chairman of the Section, presiding.

The following papers were presented at this meeting:

(1) Mr. B. A. BERNSTEIN: "On the relation between spaces in n-dimensional space and their concrete representation for the space of four dimensions."

(2) Dr. THOMAS BUCK: "Some periodic orbits of three finite bodies."

(3) Dr. S. LEFSCHETZ: "On cubic surfaces and their nodes."

(4) Professor H. F. BLICHFELDT: "On the order of linear homogeneous groups. Fifth paper."

(5) Mr. B. A. BERNSTEIN: "On an algebra of probability" (preliminary communication).

In the absence of the author the paper by Dr. Lefschetz was read by title. Abstracts of the papers follow.

1. In this paper Mr. Bernstein derives synthetically theorems concerning the fundamental relations between spaces in a hyperspace, including general formulas relating to the dimensionality of any space in any other space as element. He then shows how those relations can be represented concretely for four-dimensional space.

2. The orbits considered by Dr. Buck are obtained by the method of continuation as to a parameter which has been developed by Professor Moulton in connection with similar problems. The results are given as power series in the parameter which are known to converge for all values of the parameter sufficiently small. When the parameter is put equal to zero the solutions found reduce to the circular Lagrangian equilateral triangle solution of the problem of three bodies. For the other admissible values of the parameter the motion may be described as an oscillation about the vertices of this triangle.

3. In this paper Dr. Lefschetz gives a treatment of the cubic surface S^3 derived from a generation given by R. Sturm. The surface is obtained as the locus of the variable twisted cubic common to two quadrics which pass respectively through two other twisted cubics R_1^3 and R_2^3, having a point O in common, and which also pass through a common variable line through O. It is shown by the principle of correspondence that both curves have 6 common bisecants, not going through O, and which are therefore on S^3. The existence of the other 21 lines on S^3 and their properties are readily established. The modifications necessary in order to obtain the 21 non-ruled cubic surfaces are given except for the surface U_8 (Schlafli's classification—Salmon, Geometry of Three Dimensions, page 395), for which it is here proved that it contains no twisted cubic, being the only S^3 having this property. For the three surfaces with a node (U_6, U_7, U_8) a second construction is given, based on a modification of the construction of an S^3 by projective pencils of quadrics and planes (Steiner). In addition to their being new, the interest in these constructions consists first, in the geometric classification of cubic surfaces, according to the mutual position of R_1^3 and R_2^3 in space, and second, in making it possible to show the properties of each of the surfaces without considering them as limiting cases. This is illustrated for the case of the surface with a binode.

4. On the basis of a principle discovered by Bieberbach, Frobenius proved the following theorem (*Berliner Sitzungsberichte*, 1911, page 373): Let a finite group G of linear homogeneous substitutions contain a substitution S whose "multipliers" (i. e., the roots of its "characteristic equation," necessarily roots of unity) $\alpha_1, \alpha_2, \cdots, \alpha_n$ possess the property arc $(\alpha_i/\alpha_j) < 60°$; then all the substitutions of G of this character are mutually commutative. It follows that G is not primitive. Professor Blichfeldt's paper, based on the same principle, contains a number of theorems of which the following are noted here:

(1) If arc $(\alpha_i/\alpha_j) \leqq 72°$, then G is not primitive.

(2) No abelian subgroups of a primitive collineation group in n variables can be of order $\geqq 5^{n-1}$.

This maximum order, of importance in fixing a limit to the order of collineation groups, is presented in much lower (and correspondingly much more complex) form for large values of n. A more general theorem is given, of which the following is a special case:

(3) If the characteristic equation of a substitution of prime order p has only three distinct roots, then $p \leqq 13$, or the group is not primitive.

5. The theory of probability has underlying it both numerical and logical assumptions. Mr. Bernstein, in his second paper, presents a set of postulates for a numerico-logical theory of probability and establishes on this foundation a general method of solving the problem: Given $p_1, p_2, \cdots, p_n$, the probabilities of the events $E_1, E_2, \cdots, E_n$; to find the probability x of the event $E = f(E_1, E_2, \cdots, E_n)$.

T. M. PUTNAM,
Secretary of the Section.