mathematics from this simple study is gratifying, the improvement in geometrical imagination is valuable, and the idea of the mathematical study of essential structure is one that will go a long way in placing the correct estimate on the necessity of mathematics in a liberal education. If such a course is given by a mathematician rather than a mineralogist, and its bearing on the various problems in mathematics that it opens up is fully brought out, there are few subjects more stimulating to the student.

The text is based upon the "law of Bravais"—that crystalline structure is a finitely periodic reticulation. This law includes the law of rational indices. The first part is devoted to the study of the individual crystal, the second to the complex crystalline structures. There are two sections in the first part, the first of them being on the geometry of the crystal, the second on the physics of the crystal. In the latter the properties are classified as those due to discontinuous vectors, those due to continuous vectors. The treatment as a whole is clear and well put. The text is elementary.

<div align="right">JAMES BYRNIE SHAW.</div>

Nouvelles Tables Trigonométriques Fondamentales. By H. ANDOYER. Paris, A. Hermann. 4to. xxxii + 602 pp.

IT is not easy to arouse much enthusiasm over a table of logarithms. Most of those who use aids to calculation are apt to regard them as tools necessary to the workshop, which are taken up and laid down with absolute indifference and with absolute confidence in their complete accuracy. Perhaps the latter is justified as far as the ordinary four-, five- and seven-place tables are concerned, and if this new issue were one of the many reprints that appear almost annually it would scarcely call for notice in the pages of the BULLETIN.

But there are features of M. Andoyer's work which merit special mention. The main portion of the work consists of the logarithms of the sines, cosines, tangents and cotangents to *fourteen* places at intervals of ten seconds of arc for the whole quadrant. An additional table gives the logarithms of $\sin \alpha/\alpha$, $\tan \alpha/\alpha$ for the same interval and for the first three degrees. Four further tables used in the calculations are printed: a page of logarithms to eighteen places of certain numbers; a page of formulas for the expansions of log cos $\frac{1}{2}\pi x$ — log $(1 - x^2)$ in powers of x^2 as far as x^{40}, and similar

expansions for the sine and tangent; the logarithms of the trigonometric functions for every hundredth part of the quadrant to seventeen places, together with their variations as far as the latter are sensible; and finally the logarithms of the functions to fifteen places at nine-minute intervals, together with their variations per 10″ to the seventh order in the case of the cosines.

In undertaking the work, M. Andoyer had two courses open to him. One was the usual plan of getting the results from other tables by a more or less sustained effort at correction, interpolating where the older tables were not sufficiently subdivided. As he tells in his preface, there are only two original tables which aim at the degree of accuracy he wished to attain. The first is the Trigonometria Britannica of Henry Briggs published in 1633, the other the Tables du Cadastre computed between 1794 and 1799 under the direction of de Prony but never issued in printed form. Vlacq's well known tables were also published in 1633 but were only carried to ten places. All the tables of later date are founded on these. The tables of Briggs can be relied on to a unit in the thirteenth place, those of de Proney to the twelfth place.

The accuracy of observations has so much increased during the last fifty years that the common seven-place tables no longer satisfy the demands of those who have engaged in refined work, especially in astronomy. The problem is not so much that of getting the numerical value of a single function or of a few functions: in such cases one can usually adopt devices which grind out the result at the cost of trouble and time. Many of the present day problems are on a large scale. The calculations are turned over to professional computers, and special devices which a mathematician can adopt and use are frequently not well adapted for the computer who is only familiar with the ordinary methods. Extended tables and, if possible, mechanical devices are more and more sought after in order to economize time and money in scientific work, just as in business.

For various reasons, M. Andoyer decided to abandon the attempt to correct and extend the previous tables: there was no other course open but that of computing all the functions from the beginning. To anyone familiar with numerical work this seems an enormous task. One is astonished to read that the whole of it was done in less than two years without as-

sistance! I quote from the Preface: "Les calculs nécessaires pour l'établissement des présentes tables (sauf la Table I qui n'est pas trigonométrique) ont été faits entièrement à nouveau, par moi seul, sans aucun auxiliaire, même mécanique. Un travail régulier de chaque jour, et quelque goût naturel pour les calculs numériques m'ont permis, malgré mes occupations ordinaires et malgré quelques périodes d'inactivité ou de tâtonnements, de mener à bien ma tâche sans ennui en un temps suffisamment court, de juillet 1908 à mars 1910, soit un an et huit mois." Is there a single college or university professor in this country whose ordinary duties would leave him the amount of free time necessary to carry through in twenty months a series of calculations, the results of which occupy 600 quarto pages?

Even if M. Andoyer, whose publications in the line of celestial mechanics are well known, had occupied himself solely with the tables, the fact of their completion in so brief a time would have been worthy of notice. An examination of his Introduction reveals the methods he adopted for the purpose of abbreviating the computations. A study of these, too long to explain in a review, will repay well anyone who aspires to undertake the formation of similar tables. But it is in the methods adopted to test the accuracy of the final numbers that M. Andoyer has given the best proofs. He is not content with the common method of forming differences but has made actual use of such formulas as

$$\cos \alpha \cdot \cos \left(\alpha + \frac{\pi}{n} \right) \cdots \cos \left(\alpha + \frac{n-1}{n} \pi \right) = \pm \frac{\cos n\alpha}{2^{n-1}}$$

(where $n = 4n' \pm 1$) to test the numerical values of the functions of angles separated by considerable intervals. One may without danger agree with him when he affirms that the original manuscript is absolutely free from errors. The same care has been taken with the proofs, which have been read after the final impression; the list of errata at the end contains one error—a unit in the last place of one function,—the rest being merely defects of printing and errors amongst the signs indicating the magnitude of the remainder following the fourteenth place of decimals.

If a criticism is to be made, it is in the use of the sign just mentioned. M. Andoyer, following T. N. Thiele, puts a

+ after a number when the remainder lies between a quarter
and three quarters of the last unit set down.   This compels
anyone who wishes to use the table to insert the sign or to
provide for a possible error of three quarters of a unit in his
last place.   The ordinary method with no sign inserted leads
to a maximum error of half a unit.   It seems to the reviewer to
be preferable to use two signs, a plus, for example, when the
remainder is between a quarter and a half, and a minus with
the last figure raised one unit when the remainder is between
a half and three quarters.   These signs can then be dropped
without adjustment and with the usual error of half a unit.

    This is, however, a very small matter.   The author is to
be greatly congratulated on the successful completion of his
task, and on producing a volume which will undoubtedly be
the standard for all future tables of the logarithms of trigono-
metric functions.   The excellent typography—an important
point—should not be forgotten.   The publication has been
made with aid of a subvention from the Fondation Commercy
by the University of Paris.

                                        ERNEST W. BROWN.

Annuaire du Bureau des Longitudes pour l'An 1912.   Paris,
    Gauthier-Villars.

    THE present issue has its usual share of improvements and
additions by the alteration to more modern data of some of
the astronomical and physical constants and the inclusion
of new matter.   For a handy volume of reference in such
matters, it is probably unsurpassed by any other of the same
size and cost.   But the reviewer has had cause to wonder
whether some portions of the matter are really of much value.
This arose from his attempt to use the table of elements of
the asteroids.   On account of the convenient form in which
they are given, he incautiously adopted the elements for about
twenty asteroids, not for statistical purposes but for individual
examinations.   Two of them seemed rather remarkable and
the Berlin Astronomische Jahrbuch was searched for previous
values.   It then appeared that the periods given were er-
roneous; No. 318 was set down with a period of 2204 days
instead of 2104 days, and No. 624 with 4229 instead of 4429
days.   These are of course slips in proof-reading, but none
the less disconcerting.   In tables of continuous functions such
errors are easily seen and corrected; one cannot do this with
isolated physical constants.