for the theory based on Σ_4, and for the theory based on Σ_5, implying $J\kappa = J\breve{\kappa}$; and the property (PP_0) of being a definitely positive operation J is that $J\mu\mu$ is (P) a real non-negative number (P_0) vanishing only if $\mu = 0$.

We have specified the bases or terminologies and the postulates of the general theories F and H, and conclude this address on the foundations of the theory of linear integral equations with the expression of grateful appreciation of your so prolonged attention.

THE UNIVERSITY OF CHICAGO.

SHORTER NOTICES.

Lectures on Fundamental Concepts of Algebra and Geometry. By J. W. YOUNG. Prepared for publication with the co-operation of W. W. DENTON, with a Note on the Growth of Algebraic Symbolism by U. G. MITCHELL. New York, The Macmillan Company, 1911. vii + 247 pp.

THE book contains twenty-one lectures on the logical foundations of algebra and geometry in substantially the same form as delivered at the University of Illinois during the summer of 1909, with an appended note on the growth of algebraic symbolism. "The points of view developed and the results reached are not directly of use in elementary teaching. They are extremely abstract, and will be of interest only to mature minds. They should serve to clarify the teacher's ideas and thus indirectly serve to clarify the pupil's." "The results nevertheless, have a direct bearing on some of the pedagogical problems confronting the teacher." "Let the teacher be vitally, enthusiastically interested in what he is teaching, and it will be a dull pupil who does not catch the infection. It is hoped these lectures may give a new impetus to the enthusiasm of those teachers who have not as yet considered the logical foundations of mathematics." Such is the purpose of the author.

The first five lectures, of 57 pages, form an introduction which makes clear the nature of the problems to be discussed and the point of view from which they are approached. Euclid's Elements, a non-euclidian geometry, the history of the parallel postulate, the logical significance of definitions, axioms and postulates, and the consistency, independence and categoricalness of assumptions are here discussed. The example of a non-euclidean world, which is that of Poincaré in more detailed form, is a forceful means of showing the relation of our intuitional knowledge of space to an abstract geometry. The qualities of consistency, independence, and categoricalness receive emphasis.

The abstract but clear development of the cardinal, negative, rational, irrational, and complex numbers takes up several lectures. The notions of class, order, correspondence, group, variable, function, and limit are carefully developed and discussed. The lecture on limits deserves the special attention of geometry teachers because of its modern point of view. The axioms of Hilbert and Pieri are discussed in considerable detail. The discussion of spaces of four or more dimensions will be of interest in the light of recent popular papers published on the fourth dimension.

Throughout the lectures the historical development of the concepts considered is emphasized; this method of presentation illustrates the importance of the history of a subject to a teacher, for it shows that mathematics is a live and growing science and not fixed and finally determined. Professor G. A. Miller (*Science,* July 7, 1911) has called attention to two slight historical errors. Remarks on pedagogical principles found scattered through the book make it clear that the author is fully aware of the limitations to the use of purely abstract methods in elementary teaching. "No *formal* proof of any proposition should be attempted which seems obvious to the pupil without proof." "With all our insistence on the formal logical procedure, the important fact must not be lost sight of that formal logic is in only a small minority of cases the method of mathematical discovery. Imagination, geometric intuition, experimentation, analogies sometimes of the vaguest sort, and judicious guessing are instruments continually employed in mathematical research." Such statements show that the author retains a proper perspective while emphasizing the value of the abstract method. The following definition of mathematics forms the climax reached in the last lecture: "A mathematical science is any body of propositions which is capable of an abstract formulation and arrangement in such a way that every proposition after a certain one is a formal logical consequence of some or all the preceding propositions. Mathematics consists of all such mathematical sciences." The appended note treats from a historical point of view the three stages of rhetorical, syncopated, and symbolic algebraic notation.

The book is strongly commended to teachers and prospective teachers and will be very useful to those giving teachers' courses. While a few discussions, such as that of the first three known infinite cardinal numbers, may not be appreciated by those whose knowledge of mathematics is limited, yet practically no use is made of the technique of higher mathematics. The ability for abstract thinking required of the reader is considerable, but the careful reading of such a book cannot fail to have a broadening effect particularly upon those teaching elementary mathematics and in daily contact with immature minds. There is a need for scholarly contributions to mathematical literature in English sufficiently elementary in character to be useful and inspiring to progressive secondary teachers, and we believe the book under review to be just such a contribution.

ERNEST B. LYTLE.

Leçons de Cristallographie. Par G. FRIEDEL. Paris, Hermann, 1911. 8vo. vi + 310 pp. 10 fr.

Two things have been kept in mind by the author in writing this book. One is the purely utilitarian view of furnishing some knowledge of the subject to students of the mining college. The other is the purely cultural view of the subject. The latter might seem a little strange, considering the title. But from a mathematical standpoint, there is a distinct cultural value in the study of geometrical properties in crystallographic form. This becomes evident when we remember the thirty-two types of crystals and the related finite groups. We can assert from experience, having tried the experiment for several years, with freshman classes, that such a study is fully worth the time put upon it, even worth more than some other branches of elementary mathematics. After the class has drawn all the types of crystals, and manufactured one or more apiece, considering in each case the symmetry involved, the rotations possible, the way each face is produced from the original one, they are in a position to know something of group theory, at least in a very concrete form. A half-semester is ample time to cover the ground. The interest shown in