In § 4, the fundamental theorem of projective geometry is proved and the algebra of segments and resulting analytic geometry are developed. The fundamental theorem is proved by making use of the postulates of motion, a procedure that appears less natural from the projective point of view than it would be to choose as a postulate either this theorem itself, or the theorem of Pascal.

The projective development is then applied for the derivation of the fundamental metric forms of non-euclidean geometry. The equations of lines are obtained, and the trigonometric functions sine and cosine are defined projectively. The theory of triangles then follows readily. The characteristic constant k occurring in these formulas depends on the character of the absolute involution obtained on each line. Congruence of triangles is developed, and construction of the triangle from given parts. The proof of the independence of the parallel axiom is made by exhibiting in the classical way the analytic number spaces for which each hypothesis concerning parallels is valid. The relation of the parallel axiom to the angle-sum of a triangle and to motion is also discussed. In § 7, plane geometry is developed without the use of 3-space, following the methods of Hjelmslev. In § 8, the role of the Archimedean axiom is treated. The proof that while the Pascal theorem does not follow from the projective postulates alone, it does follow from these with the addition of the Archimedean postulate, is especially interesting.

As a whole the book is a valuable addition to the literature of the subject. More references to original sources would add much to its value. It is not free from typographical errors, but those noted are fairly evident from the context.

F. W. OWENS.

Précis de Mécanique rationnelle. Introduction à l'Étude de la Physique et de la Mécanique appliquée. Par P. APPELL et S. DAUTHEVILLE. Paris, Gauthier-Villars, 1910. 8vo. 729 pp. 25 francs.

To condense an enormous Traité in three volumes and over 1,800 pages into a *précis*—a very significant term—of one volume and about 700 pages, demands a treatment which is more than a mere deletion of certain parts. The whole must to some extent be recast. Everyone has seen abridged treatises which were worthless and by the side of the original quite obscure. In the present case however the task has been successfully done, and we have in this short course an admirable treatise on mechanics which covers a very wide range in a thorough manner. As the title indicates, the subject considered is rational mechanics, yet it is brought so close to applied mechanics that the passage from one to the other is very easy. Although the standpoint of the text is modern, yet it is above all a book for the undergraduate student.

The object of mechanics is stated to be the solution of two problems:

To find the motion of a system of bodies under the action of given forces;

To find the forces that will produce a given state of motion. By force, it is carefully explained, is meant a purely fictitious cause, not a real physical cause. It is merely an abbreviation for the product of acceleration by mass. Mass is defined to be the arbitrary number that may be assigned to a material point, such that the inverse ratio of two such arbitrary numbers is equal to the ratio of the accelerations they produce in each other. Thus all problems are reduced to questions of acceleration, that is, to questions of functions of four variables x, y, z, t and the derivatives of the first three as to the fourth. In the preface of the second edition of the third volume of the Traité, attention is called to a note at the end of the volume on "la théorie de l'action euclidienne." The object of this note is to reduce the whole theory of mechanics to the consideration of certain invariants in euclidean displacements. The note is not repeated in the Précis, but the spirit of it is in evidence throughout. The development of the course, however, does not demand on the part of the student a familiarity with group theory nor even great knowledge of analysis. Some acquaintance with differential equations is necessary. Intuitive methods are used to some extent and particular problems are developed in detail usually, in order to make the use of general theorems quite clear. The ripe experience of the authors shows itself continually, so that one feels that he has here a finished product of many years preparation. We will undertake to indicate some of the main features in some detail.

There are three parts: Preliminary Notions, Statics, and Dynamics. In the first we find chapters on vectors, kinematics, and the principles of mechanics; in the second, on equilibrium of points, of solids, of deformable systems; in the third, on dynamics of a point, moments of inertia, the seven universal equations of motion, motion of a solid, friction, impact, virtual work, d'Alembert's principle and Lagrange's equations, attraction and potential, hydrostatics, and hydrodynamics. A selection of examination problems closes the book. The omissions are in general of the more elaborate problems or the more difficult parts of the theory.

Although, as seems to be the more common custom in France, the notation of vectors is not used in this course, nevertheless the first chapter is devoted to a sketch of the different kinds of vectors and their more important theorems. Three classes are noted: the free vectors, the sliding (glissant) vectors, and the bound vectors. The terminology of these is freely used throughout the course. The first class include those which combine according to the parallelogram law, such as translations, velocities, accelerations, axes of couples, and like quantities. The second class comprise such quantities as forces acting on a rigid body, linear moments, moments of momentum, screws, etc. The third class are the localized vectors, such as a velocity field, acceleration field, electromagnetic field, vortex field. The distinction between axial and polar vectors is also explained. Two invariants are noted: $X^2 + Y^2 + Z^2$ and $LX + MY + NZ$. The first is the square of the length of the resultant, the second represents the sum of the volumes of all the tetrahedra made by taking the sliding vectors in pairs as the opposite edges of tetrahedra.

The chapter on kinematics has been doubled in length over that in the Traité. This is partly from the examples introduced, but it is also due to the very much more detailed treatment of the definitions and the theorems. There is a distinct gain for the elementary student. The development produces the usual expressions for the velocity and the acceleration in various coordinates, discusses the motion of a solid, and the problems of relative motion. If one were to offer a suggestion on this part of the book, it would be that a brief consideration of relativity in general might not be amiss.

The chapter on the principles of mechanics lays down the following as the foundation of what follows:

1. Inertia: every material point which is supposed to be isolated has no acceleration.

2. Two material points moving with any velocities produce in each other oppositely directed accelerations along the line

joining the two points. (The action may be due to electricity, gravity, magnetism, etc.)

3. The ratio of the numerical values of these accelerations is constant, that is, the ratio is independent of the physical conditions of the points, or of their motion. These ratios are inversely as the ratios of the masses, one mass being purely arbitrary. Masses are analogous to chemical equivalents.

4. The accelerations of a point due to several points compound according to the parallelogram law.

One might suggest that here a brief note or paragraph could have been given on the definitions of mass which depend on the velocity, and the place of the "new mechanics" with regard to the "Newtonian mechanics." Such an explanation would orient the student as to some of the more recent work in mechanics.

The notion of force function and field of force are introduced in connection with the idea of work. The principal fields are mentioned: uniform field, gravity field, electric field, magnetic field.

The general conditions of equilibrium are stated in the succinct form: That any system be in equilibrium it is necessary and sufficient that the forces form a system of sliding vectors equivalent to zero. In the discussion of the equilibrium of a solid body we miss the single condition given in the Traité: for the equilibrium of a solid body the necessary and sufficient condition is that the sum of the moments of all the forces with respect to each of the edges of any tetrahedron vanish for each edge. The simplicity of this vector statement can scarcely be improved. It is also very general, as the tetrahedron may be chosen to suit the investigation. The discussion of the subject of the chapter is more detailed than in the Traité, with an increase of clearness and simplicity. The consideration of virtual work has been transferred to a later chapter.

Chapter VII, opening the third part, is devoted to the dynamics of a point. These topics are discussed: general theorems, straight motion, curvilinear motion of a heavy point in a vacuum and in a resisting medium, electrified particle in a field, motion of a point on a surface, equilibrium and relative motion. The text follows the Traité, the more special or complicated cases being omitted. The illustrative examples are carefully worked out and discussed.

The chapters on Lagrange's equations, d'Alembert's principle, and canonical equation have been placed farther on, after the treatment of virtual work.

Chapter IX considers the dynamics of systems, general theorems, and the seven universal equations of motion. These theorems include the conservation of the motion of the center of gravity of the system, of the sum of the moments of momentum, and of vis viva. These lead to seven general equations of motion, three for the center of gravity, three for the moments of momentum, one for the kinetic energy. The internal forces do not enter the first six, but are present in the last. It is shown that these equations remain unchanged if the coordinates are referred to axes moving parallel to themselves with a uniform motion, or to axes moving parallel to themselves and always having the center of gravity as origin. It is pointed out that the motion in any case is reducible to the motion of the center of gravity plus that of the parts of the system with reference to the center of gravity. Thus the question of relative and absolute motion enters again. The meaning of energy and the law of conservation of energy close the chapter.

Chapter X takes up the motion of a solid body. The discussion is divided into the study of motion about an axis, motion parallel to a plane, screw motion, motion about a fixed point, motion about a point in a field which is directed to the point, motion about a fixed point in a gravity field, and motion of a free solid. The particular problems worked out are the composite pendulum, a homogeneous straight wire passing through a fixed point and sliding in a plane, a heavy plate sliding on a line in a plane. The Traité considers several other problems, some of which might have come in here: the Atwood machine, wheel rolling down an incline, double cone, the hoop, the top.

Friction is considered in the following chapter. It is practically defined as the effect of a force tangential to the surface, and two couples, one whose axis is the normal, and one whose axis is tangent. There are thus friction of sliding, of spinning, and of rolling. The spinning friction is omitted from the discussion.

Chapter XIII is devoted to developing the principle of virtual work which gives rise to what is here rather technically called analytic mechanics. From this principle are deduced the general conditions of equilibrium, and its importance is pointed out for that treatment of dynamics which reduces the problems to problems in statics. The next chapter takes up the principle of d'Alembert, which is made the sole basis of the development. The principles of Gauss and of Hamilton and the principle of least action, are given in the Traité but omitted here. Lagrange's equations, the distinction between holonome and non-holonome systems, small movements about a position of equilibrium, and the canonical equations are discussed. These are all illustrated with examples worked out in full and with practical directions as to the use of the theorems. It is also suggested that many of the problems could as well have been solved by methods previously given. The next chapter completes the treatment of an earlier one on impact. A method analogous to the principle of d'Alembert is used.

In Chapter XVI we find Green's theorem, stated in the usual way and also in the vector form: the integral of the flux of the field through a closed surface is equal to the integral of the divergence of the field over the points enclosed by the surface. This theorem is then applied to the theory of attraction and potential.

The last chapters are concerned with the equilibrium of fluids, and with hydrodynamics. Both Lagrange's and Euler's methods are developed, and there is a brief treatment of vortices.

Taken as a whole, it is difficult to see how the text could be better. The book has a finished character, and is so practical as a text that one lays it down well pleased with its form and contents.

JAMES BYRNIE SHAW.

Systèmes cinématiques. Par L. CRELIER. Paris, Gauthier-Villars, 1911, 99 pp.

THIS monograph is one of the recent numbers of the well-known French collection Scientia. The author confines himself to the solution of a number of problems in generating plane algebraic curves by link motions, so that the general title of kinematic systems for the ground covered, even when limited to the plane, seems not entirely justified.

We should expect an introductory chapter on the general principles of link motions and the transformations realized by them. The theorems of Kempe and Koenigs concerning the realization of all algebraic curves and surfaces by linkages should at least be mentioned. A treatment of general propositions like these would be of far greater importance