(4) The jack-screw problem is said to be "a problem based on facts." Granted. Also "it was . . . not intended to be a complete and actual problem at all," and it is implied that one would be "lacking in common sense" "to consider such a computed result as finding actual application in the practice of raising buildings." Exactly. That was why it was chosen for criticism. The book aims to show the pupil "what the shop problems are."*

(5) The reviewer is accused of "gross misstatement" of the problem on spiral gearing and malicious intent on his part is implied. The reviewer would point out that of the four results above mentioned, one, the lead of the spiral, is not even mentioned in the solution of the problem as given by the author of the book. The other three quantities may be determined by the formulas: number of teeth $= N = Dp \cos Y$, blank diameter $= D + 2/p$, circular pitch $= \pi/p \cos Y$. Whether any single one or all three of these quantities are to be found from the given data, the determination does not involve operations equivalent to multiplication followed by division by π. This is true also of the determination of the dimensions of the driven gear, and of the numbers of the milling cutters to be used, both of which are given in the book, though not mentioned above. The question at issue is not one of "short cuts" in computation, but of comprehension of the principles and formulas underlying the computation. The reviewer sees no cause to retract any portion of his criticism of the problem.

CHARLES N. HASKINS.

SHORTER NOTICES.

Arithmétique générale. Par EMILE DUMONT. Paris, A. Hermann et Fils, 1911. 8vo. xvii+275 pp. 10 francs.

THIS volume presents a treatment of certain number fields and the ordinary laws of operation within these fields. The book is divided into four parts, of which the first treats of

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* Introduction, p. viii.

natural numbers, the second of qualified numbers (nombres qualifiés), the third of complex numbers, and the fourth of quaternions and ternions. The treatment is elementary and no attempt is made to extend the subject or to present novel results. The preface, which by the way is one of the most interesting parts of this book, characterizes it quite fully and justly. We quote: "The definitions and the properties of numbers are generally studied by the young student at quite different times and from different classical works. . . . In all this there is no unity of method or point of view. . . . I have therefore attempted to furnish a treatment which should be of use to him who has finished his preliminary studies and who wishes to take a retrospective view of mathematical principles before he launches himself into the vast fields of mathematics and physics."

Mathematicians are classified as "logicians" and "rationalists." "In analysis the logicians reason on the written characters and in geometry they reason on words." "The rationalists consider mathematics as a preface to physics. They are constantly guided by reason based on experience."

The author follows throughout what he regards as the program of the rationalists. In his hands the fundamental proposition of mathematics (axioms, primitive propositions, or whatever we may call them) are corollaries of physics or of intuitional geometry. Numbers are used to express relations between "magnitudes." These "magnitudes" have properties which we obtain from direct experience with them and these properties we express numerically. The general properties of numbers are determined by the service which they are required to render. The notion of axioms and undefined symbols finds no place in this treatment. There in not an unproved proposition in the book. All the statements which we are wont to regard as assumptions appear under the caption of theorems. The proofs consist in references to remarks on the nature of magnitudes.

From the point of view of the rigorist this treatment is impossible. The reviewer must confess however that there is a certain charm about its naivété. It is refreshing to see this attempt to set mathematics directly and consciously into life and to validate its elementary propositions directly from a wealth of experience. When the true function of the rigorist is more generally understood we shall perhaps have more patience with this sort of mathematics. With the rigorist the question at issue is frequently not whether the proposition which apparently is being considered is *true* or not, but whether it follows from some other proposition. That is, how much must we say to include by implication a certain other body of propositions.

It seems equally legitimate to inquire what must be the properties of systems of numbers in order that they shall express conveniently and accurately the varied phenomena that daily impinge upon us.

The prospective reader must judge for himself whether this is the sort of book he wants to read. Does he wish to study the modern development of the various algebras as based upon definite assumptions? Then this book is of no use to him. Does he wish to see an attempt to develop these algebras as corollaries of physics? Then it is probably the best book he could find.

N. J. LENNES.

Grundlagen der Geometrie. Von Dr. FRIEDRICH SCHUR. Leipzig, Teubner, 1909. vii + 192 pp.

THE Neuere Geometrie of Pasch marked the first effort to set up a complete set of fundamental statements for geometry —if we except the manifoldness development of Riemann. Following the appearance of that book there came a remarkable development of the subject that has thrown great light upon the logic of geometry. Schur wishes the present book to be considered as, in a sense, a revision of Pasch. Although the author expresses very strongly his indebtedness to Pasch, in only the most general way can the book be said to be such a revision.

In the general trend of his development the author follows Peano, in that congruence is obtained from motion or from projective geometry rather than directly from postulates, as is done by Hilbert, for example.

The general problem is formulated as follows: "To set up a simple and complete system of intuitive facts or axioms, entirely independent of one another, from which geometry can be derived by purely logical processes. To deserve the name geometry, axioms must be employed which express the results of the simplest and most elementary consideration of geometric figures, from which they are obtained by abstraction." This of course bars out the idea of space as a number manifold.