Notice sur le Système des six Coordonnées homogènes d'une Droite et sur les Éléments de la Théorie des Complexes linéaires. Par A. SÉFÉRIAN. Lausanne, A. Dénereaz-Spengler et Cie., 1910. 79 pp.

To quote from the preface: "At the end of the introduction to his work Statique graphique, Professor Mayor supposes the reader familiar with line coordinates and the theory of the linear line complex. The present work develops this theory and has for aim the preparation of the reader to read profitably Professor Mayor's book."

A point (x, y, z) and the direction cosines (X, Y, Z) of a line through it uniquely fix the latter. The projections (L, M, N) of the moments $L=yZ-zY$, etc., upon the coordinate planes furnish three further equations, from which x, y, z can be replaced by L, M, N. The six larger letters are the coordinates under discussion. Various theorems are now developed regarding relations between vectors and their projections, all results being expressed in terms of the coordinates.

The moment of two vectors is derived; it vanishes when the lines defining the vectors intersect. Linear systems are considered, pencils, bundles, reguli, all of them in vector language and, in part, vector notation. The moment of two linear systems furnishes a natural introduction to the concept of the linear complex. The ideas developed are polarity, conjugate lines, condition for involution, and the invariance of the anharmonic ratio under linear point transformations and duality. Finally, various points of contact with Professor Mayor's book are pointed out. The work does not pretend to be a mathematical contribution, but to contain a mechanical interpretation of well-known ideas, without any of the purely geometric applications.

VIRGIL SNYDER.

Theoretical Mechanics. By PERCEY F. SMITH, and WILLIAM RAYMOND LONGLEY. Boston, Ginn and Company, 1910. 288 pp.

THIS is an attractive volume of 288 pages in a binding similar to the other mathematical texts edited by Professor Smith. The pages have a neat appearance, the size of type and spacing being excellent, and the numerous diagrams throughout the book could scarcely be improved.

From the preface one learns that the text is intended primarily for the mathematical student and a perusal of the volume confirms the statement. The analytic side of the subject is kept constantly in the foreground while the geometric or intuitional side is wholly in the background. Thus the mathematical student will find here an excellent field for applying the ideas which he gathered in the calculus. Integrals of various kinds and differential equations of certain types are abundantly in evidence.

The subject matter of the volume is almost exclusively dynamics and it would seem that "Analytical dynamics" would have been a more accurate title than "Theoretical mechanics." Of the thirteen chapters eight deal with the motion of a particle or a system of particles, one with the motion of a rigid body and two with work and energy. Thus with the exception of the first and last chapters the entire book is devoted to kinematics and dynamics.

The first chapter is devoted to the integrals arising in the computation of centers of gravity, moments of mass, and moments of inertia. The authors make the point that a student fresh from the study of the integral calculus can dispose of these integrals before the real study of mechanics is begun. From a purely mathematical point of view this is true, for one can evaluate a set of integrals quite independently of whether or not there is any physical concept underlying them. From a mechanical point of view, however, it is not true, for the underlying physical concept is the main point. It is rather awkward to open a book with a reference to future chapters, and "The student is asked to accept these formulas as *definitions*" has quite the flavor of an apology. If this first chapter were the last chapter in a text-book of calculus the necessity for this would be apparent, but to be compelled to apologize for a lack of mechanical ideas in a text-book on mechanics merely shows that the subject matter of the chapter should not be at the beginning of the book. It is in the wrong place.

The second chapter takes up the subject of the motion of a point along a straight line. In the third chapter, on the curvilinear motion of a point, the conception of vectors is introduced and a very good discussion is given. The fourth chapter introduces the conception of force. The first of Newton's laws of motion is stated, the second is described, while the third is reserved for the following chapter. The description of the second law is not accurate. It says "change in momentum is caused by forces acting upon the body," and again "Force is therefore the cause of acceleration." As a matter of fact, Newton's second law says nothing about causation. It merely defines what is to be understood by "force" and how it is measured.* Axioms I and II, page 100, were considered by Newton as corollaries to the second law and not as independent axioms. In the fifth chapter, entitled "Work, energy, impulse," one finds a short discussion of constrained motion which requires the statement of Newton's third law. This subject seems to belong rather to the preceding chapter. Had it been placed there the three laws of motion could have been brought together and their fundamental importance in mechanics properly emphasized.

Chapters six, seven, eight, and nine deal respectively with motion under constant forces, central forces, harmonic motion, and motion in a resisting medium. These chapters are well written. Chapter ten, on the potential and potential energy, is short but clear. One would rather expect chapter eleven, on systems of material particles, to contain an application of the potential, but such is not the case. The idea of the potential is developed but is left barren. The twelfth chapter, on dynamics of rigid bodies, contains a relatively small number of theorems and a relatively large number of problems. Indeed we find nine pages of text and six pages of problems.

Chapter thirteen, the last chapter in the book, contains a few theorems in statics. It is wholly inadequate. Its deficiency in this respect is well illustrated by the fact that so important a concept as a couple is nowhere mentioned. Perhaps the reason for this scant treatment of statics is the fact that statics does not lend itself to such beautiful mathematical developments as does the subject of dynamics. A mathematical student seeking exercise for his mathematical faculties and material for his mathematical ideas will find in this book a selection of topics well suited to him and a superabundance of problems. But a student of mechanics or physics who is interested in obtaining a concrete hold on mechanical principles

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*On this point Poincaré has made the following remark: "Quand on dit que la force est la cause d'un mouvement, on fait de la métaphysique, et cette définition, si l'on devait s'en contenter, serait absolument stérile. Pour qu'une définition puisse servir à quelque chose, il faut qu'elle nous apprenne a *mesurer* la force; cela suffit d'ailleurs, il n'est nullement necessaire qu'elle nous apprenne ce que c'est que la force *en soi*, ni si elle est la cause ou l'effet du mouvement." La Science et l'Hypothèse, p. 120.

and is not interested in the mere mathematics of the subject will probably find the book unsuitable.

Some misprints have been noted, but they seem to be few. The footnote on page 99, "A finite periodic function of the time must have the form $A \sin (bt+\nu)$ or $A \cos (bt+\nu)$, where A, b and ν are constants," is a decidedly curious statement.

W. D. MacMillan.

Die partiellen Differentialgleichungen der mathematischen Physik. Nach Riemann's Vorlesungen in fünfter Auflage bearbeitet von Heinrich Weber. Erster Band. Braunschweig, Vieweg und Sohn, 1910. xviii+528 pp. Unbound, M. 12; bound, M. 13.60.

The earlier editions of this work have won for themselves an important place in every mathematical library, where they are frequently consulted both as reference books and for more consecutive reading supplementary to courses on mathematical physics. They have thus become so well known that an extensive review of the volume before us would be superfluous. Professor J. S. Ames has ably reviewed the fourth edition in this Bulletin (volume 8, page 81). The following comment will therefore be limited to a comparison of the fourth and fifth editions.

In the preface to the fifth edition, Professor Weber remarks that some of the developments of mathematical physics during the past decade have been too significant to leave unmentioned, and that although they have not been carried sufficiently far to permit of finished exposition in a text-book, they should receive some attention in the pages to follow. He speaks of integral equations and the notion of relativity as two of the more important contributions made to the science since his last edition in 1900, and promises an application of the first in the present volume. The failure to find any satisfactory fulfillment of this promise will be the reader's greatest disappointment. In its place there appears a brief exposition of Neumann's method of the arithmetic mean, with the old assumption of a convex surface, a condition made unnecessary by the use of integral equations. A suggestion is indeed given for the removal of this restriction, but it is not developed further, and appears inadequate. It is not even pointed out that an integral equation is being solved, whereas, as Professor Mason has shown in his New Haven Colloquium lectures, Neumann's