

carry through this proof the general solution of

$$y'' + \lambda A(x)y = f, \quad y(a) = y(b) = 0$$

is first derived. The minimum problem which is to be treated is to minimize $\int_a^b y'^2 dx$ under the condition that $\int_a^b A u^2 dx = 1$ and $\int_a^b y_i y dx = 0$ ($i = 1, 2, \dots, n$), where y_1, \dots, y_n are the solutions belonging to $\lambda_1, \dots, \lambda_n$. This gives in order of increasing magnitude the positive values of λ ; these exist in infinite number if ϕ is anywhere positive. Likewise a series of negative values of λ will be obtained if ϕ is anywhere negative.*

A formal expansion of an arbitrary function is then given by

$$f = \sum_{-\infty}^{+\infty} c_i y_i, \quad c_i = \pm \int_a^b f A_i y_i dx,$$

and Professor Mason states the theorem that this expansion holds if f vanishes at a and b , is continuous, and has a derivative continuous save at a finite number of points. The proof however contains an error. †

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SHORTER NOTICES.

Vorlesungen über Algebra. Von GUSTAV BAUER. Herausgegeben vom Mathematischen Verein München. 2te Auflage. Leipzig und Berlin, B. G. Teubner, 1910. vi + 366 pp.

THAT Professor Bauer's lectures, which were published in honor of his 80th birthday by the Mathematical Club of Munich in 1903, are destined to outlive their author by many years, seems to be evidenced by the fact that a second edition became necessary in 1910, seven years after the first edition and four years after Professor Bauer's death, which occurred on April 3, 1906.

*See an article in the *Transactions*, vol. 8 (1907), p. 373.

† At bottom of p. 219 it is necessary to replace the multiplier 2 in the inequality

$$(\phi_{n,m})^2 \leq 2 \sum_{i=n}^m c_i^2 \left(\int_a^x y_i' dx \right)^2$$

by a multiplier $m - n$. This error appears to destroy the force of the proof.

A fine spirit of veneration for the deceased teacher and friend breathes from the preface for this second edition, written by Professor Karl Doehlemann, who has now taken full charge of the book. It is clear that Professor Bauer, whose portrait appears opposite the title page, was much respected by those who worked under him and with him.

As the first edition of these lectures was quite fully reviewed by Professor Dickson on pages 257–260 of volume 10 of the BULLETIN, the task of the present reviewer may be limited to a discussion of the points of difference between the two editions. In Part I, devoted to the general properties of algebraic equations, the most important change consists in the substitution of Ulherr's proof of the fundamental theorem of algebra for the proof of Argand, which only showed that if $f(z) \neq 0$, a complex number h could be found, such that $|f(z+h)| < |f(z)|$. Ulherr's proof makes use of the fact, that when z describes a circle about the origin in the z -plane, $w = f(z)$ describes a curve in the w -plane which goes about the origin n times, if $f(z)$ is of degree n .

The notion of a domain of rationality, formerly taken up in the chapter on the Galois theory, is now introduced in Chapter IV, as are also the notions of reducibility and irreducibility of functions and of equations.

Part II, which treats the algebraic solution of equations, has undergone greater changes than the other parts. The chapter on the Galois theory of algebraic equations has been omitted and a new chapter has been inserted on Abel's theorems concerning the general representation of algebraic functions. This makes possible a more rigorous presentation than was formerly possible of the Abel-Wantzel theorem that equations of degree higher than the fourth are not in general solvable by radicals. In this new Chapter XV, which follows exactly the lines of the classical treatise of Serret, one feels a dissimilarity from the older chapters in the complete absence of illustrative examples, so abundant elsewhere. The book, as stated in the preface, does not take the ultra-modern point of view, and is intended for students who for the first time are making a systematic study of the science of algebra. In this chapter, where they are getting a first introduction to the more difficult parts of the subject, a few well chosen examples would help them to keep the general statements connected with the concrete situations of which they are the abstraction.

Part III, in which the numerical solution of algebraic equations is taken up, is left practically unchanged.

In Part IV, theory and application of determinants, the chapter on linear equations (Chapter XXVI) has been almost completely rewritten by Dr. Perron. The subject is now presented more systematically, by the reduction of the solution of a system of n linear non-homogeneous equations in n variables to the solution of a system of n linear homogeneous equations in $n + 1$ variables. By introducing the notion of rank of a matrix (formerly postponed to the last chapter in the book) the results are stated more compactly. Although the treatment of this subject has thereby become much clearer, it seems to the reviewer that still further progress towards unifying it can be made, as is shown for instance on pages 43–53 of Bôcher's *Introduction to Higher Algebra*.

Probably in order to remove the objection to the proof of the multiplication theorem as given in the first edition, raised, for instance, in Professor Dickson's review, the theorem is now introduced that a determinant is an irreducible function of its n^2 elements whereby the difficulty referred to is removed.

One regrets the scarcity of geometrical illustrations and interpretations. The numerous examples illustrating the methods presented make the book valuable for the student. It seems strange that the treatment of determinants should be left for the last section of the book; if presented earlier, the subject of elimination, which is fully treated and is now taken up in Part I and again in Part IV, could be discussed more compactly. The presentation is on the whole very clear; it goes step by step and leads gradually to a full understanding of the topics discussed, which furnish a good introduction to the study of algebra.

It is remarkable, that while the typographical errors of the first edition have been corrected, nearly as many new ones have found their way into the second edition.

ARNOLD DRESDEN.

Théorie mathématique des Assurances. Par P. J. RICHARD et E. PETIT. Paris, Doin, 1908. 396 pp.

THE series of books coming out under the general title *Encyclopédie Scientifique* seems not to be very well known in this country, yet the enterprise is highly ambitious and indus-