really unfamiliar with the subject it would be well to read Wien before Lorentz and then again afterward. It is an excursion that is worth while for anyone. Come, drop an epsilon just for once and pick up an electron; it is a deal larger, even if its radius is only 10^{-13} cm.

EDWIN BIDWELL WILSON.

MASSACHUSETTS INSTITUTE OF TECHNOLOGY,
BOSTON, MASS., *August*, 1910.

SHORTER NOTICES.

Vorlesungen über Differentialgeometrie. By R. v. LILIENTHAL. Erster Band: *Kurventheorie.* Leipzig, B. G. Teubner, 1908. 8vo. vi + 368 pp. 12 Marks.

IT is undoubtedly one of the achievements of the mathematical development of recent years to have shown that mathematical rigor is not the same as pedantry; that a theory may be presented in an irreproachable manner and still be attractive and interesting. There can be little doubt from a pedagogic point of view that if, in a treatise intended for students, a choice must be made between a comprehensive but tedious, and a less comprehensive but interesting treatment, the latter should be given preference, provided of course that its limitations be properly indicated. Both of these general principles seem to have been ignored completely in the construction of the book under review.

The author wishes to be rigorous and general. For this purpose he excludes all considerations involving infinitesimals. One might begin to quarrel with him on this score, since a properly formulated notion of infinitesimals is not altogether unknown. He then confines his attention exclusively to the case in which all of the functions which occur are analytic, a restriction of generality which seems extraordinary from the author's point of view, since all of his developments require only the existence of a finite number of derivatives. But even then, also for the sake of rigor, he excludes (as a matter of course, he says), the consideration of questions of order of contact, although all of these questions are easily treated by power series methods and would fall most naturally into the theory as developed by him.

It is true, historically, that differential geometry has up to the present time been occupied primarily with metric questions. We should doubtless also exercise a certain amount of patience with those who are accustomed to think of differential geometry as exclusively metric, and who refuse to bother with projective considerations in their own work. Still, in a comprehensive treatise on differential geometry, either the projective theory should be included, or else the exclusion should be explicitly indicated by the introduction of some qualifying adjective into the title; we should suggest *metric* differential geometry. The author does, however, mention briefly one phase of projective differential geometry by discussing the curves which belong to a linear complex. The following amusing remark is of interest, however, in showing how far removed he is from the modern spirit of geometry, in which the change from one space element to another has turned out to be so exceedingly fruitful. He says: "Die Plücker'sche Theorie ist entweder ein Beispiel dafür, dass rein formale Überlegungen zu wichtigen geometrischen Begriffsbildungen führen können, oder sie ist, erst nachdem sich ihr Urheber auf anderem Wege von der Bedeutung des Komplexbegriffes überzeugt hat, unter dem Vorbilde der gewöhnlichen Theorie der Ebene und der Flächen zweiten Grades aufgestellt." He obviously means, as the context shows, that the importance of the notion of a complex consists entirely in its applicability to kinematics.

Another omission which, to the reviewer, seems a serious one, is that no mention is made of intrinsic geometry as developed principally by Cesàro. The general theorems which are connected with this point of view, and the added strength which it gives to the investigator, are so valuable that no treatment of the theory of curves which goes beyond the elements can be considered adequate which does not at least give some account of this theory.

In spite of these criticisms, there are a number of things in the book which deserve high commendation. The treatment of one-parameter families of plane curves is very careful, and involves some new points of view, although the unusual distinction between envelope (Einhüllende) and curve of contact (Berührende) is of doubtful value.

The ordinary treatment of the differential geometry of plane curves is essentially a geometric interpretation of the first and second derivatives by means of the tangent and the circle of curvature. A geometric interpretation of the third derivative is due to Abel Transon (1841) who introduced the notions abberrancy of a curve, and axis of aberrancy. This notion has been almost completely lost, and the author is to be commended for reviving it. Transon's term, however, was "axis of deviation," which just reverses the historical statement as given by the author in regard to these two names.

The book is carefully printed and none of the misprints noticed by the reviewer can give rise to any difficulty.

E. J. WILCZYNSKI.

Elliptische Funktionen. Von Professor Dr. KARL BOEHM. Erster Teil. Göschen (Sammlung Schubert XXX). Leipzig, 1909. xii + 354 pp. 8.60 Marks.

A TREATISE on any of the functions of analysis, the properties of which are well known, must rely for its usefulness upon the mode and style of presentation. The historical development may be followed, or the functions may be introduced through later discovered properties. There can be no doubt that the former is the natural and more easily comprehended introduction, especially to the higher functions. Professor Boehm has elected the latter course, in this first volume, with the understanding, however, that the reader may commence with the second volume which starts out with elliptic integrals and the inversion problem.

The present volume is occupied with the various infinite series which represent simply and doubly periodic functions, with related series and products, and with their mutual interdependence.

The beginner will probably do well to take the author's suggestion and commence with the second volume. Students who have had a good course in the calculus can easily appreciate the inversion problem and its close proximity to so-called applications, but would most likely become discouraged and cry cui bono if requested to assimilate the contents of this volume in order to become acquainted with elliptic functions. This is true even if the shorter course were followed which the author has carefully planned and indicated by footnotes at the proper places through the volume.

It must be said, however, that the volume contains all the necessary preparation for an understanding of the series repre-