SHORTER NOTICES.

Vorlesungen über Differential- und Integral-Rechnung. Von EMANUEL CZUBER. Zweite, sorgfältig durchgesehene Auflage. II Band. Leipzig, Teubner, 1906. 8vo. viii + 532 pp.

THE present book is the second volume of a course of lectures prepared by the author primarily for students in technical schools, but with the hope that it will meet the needs of students in a "narrower" sense. The first edition of the two volumes appeared in 1898 and met with a success that called forth this second edition eight years later.

The course was prepared, as appears in the preface to the first edition, in the firm conviction that the calculus has a twofold mission to perform. On the one hand, it marks the close of the mathematical training of most students in the technical school, and should therefore contain sufficient material to give a scientific conception of technical problems, and an ability to read intelligently the rich literature in this domain. On the other hand, it is the gate through which the student who elects the study and teaching of mathematics as his life work must enter his chosen field. It must therefore contain enough of the spirit of research to give the student a mental training in, and appreciation of, modern analysis and rigor, in so far as this is possible with the material chosen for exposition.

In the present volume the student is introduced at once to the fundamental problem of the integral calculus, *i. e.*, the solution of the equation

$$\frac{dF(x)}{dx} = f(x),$$

and a clear distinction is made between the *formal* solution derived from the mean value theorem of the differential calculus and a *practical* solution.

The definite integral precedes the indefinite integral, and in the first section of Chapter I one finds its definition and usual properties together with a good definition of an integrable function leading up to the *principal theorem* of the integral calculus,

$$\int_a^b f(x)dx = F(b) - F(a).$$

The remainder of the chapter is devoted to the formulas for immediate integration and the usual methods of substitution and integration by parts.

In Chapter II is found the integration of rational functions, irrational functions, and transcendental functions. The treatment could not well be otherwise than usual. One finds, however, throughout a clearness of statement and an insistence upon the distinction between a theoretically complete solution and solutions dependent upon isolated methods.

Chapter III contains the treatment of simple and multiple definite integrals. The simple definite integral has already been defined (Chapter I) and here we find methods for its evaluation together with the mean value theorems and an excellent treatment of improper integrals, not only in the cases where the indefinite integral can be expressed by foregoing methods, but also in cases where this is not possible, provided the function to be integrated does not change sign in the interval (x_0, b) where $a \leqq x_0 < b$ and a and b are the limits of integration $(b \leqq \infty)$. In case the function does not satisfy this condition, a statement of the method to be used in testing for the existence of the integral is given and two illustrative examples worked.

The use of infinite series for evaluating integrals is preceded, as it should be, by the theorem upon the integration of infinite convergent series.

There follows a treatment of the integral as a function of one of its limits; also as a function of a parameter. It is interesting to note that the double definite integral is led up to by means of integration under the integral sign, and the consequent proof that the value of the double iterated integral * between constant limits is independent of the order of integration. Double definite integrals are carefully defined and satisfactory proofs given that they can be evaluated by means of iterated integration. Triple integrals, and multiple integrals in general, follow without unnecessary repetition of proofs. Improper double integrals are defined and illustrated by examples. The chapter closes with a section devoted to eulerian integrals and Fourier series. This section has been added to the first edition of the book and materially increases its value as an exponent of the two-fold mission stated above.

The application of the integral calculus to geometry, me-

* Czuber uses the term " Zweifaches Integral." P. du Bois-Reymond and O. Stolz use the expression " Zweimaliges Integral."

chanics, and physics has been deferred to the fourth chapter. The student, having been furnished with the tool and an appreciation of its power, as well as its limitations, is ready to use it. The calculation of moments of inertia and centers of mass as well as Green's theorems have been added to the first edition. This, together with the addition noted above, marks practically the only change from the earlier edition of the integral part of the course. The sections on applications to mechanics and physics are especially well written. The student who goes over the book thus far will have an intelligent conception of the problems in mechanics to which the integral calculus is especially applicable and the use of Laplace's equation and Poisson's equation, as well as a knowledge of the meaning of such terms as "fields of force," "force function," "tubes of force," "strength of field," etc.

The fifth and last chapter is devoted to differential equations and covers nearly one-third of the book. The effort being to emphasize the integral calculus as an instrument to be used by the student in dealing with technical problems, this is as it should be. At the same time the finer use as a mental training in mathematical thought is not lost sight of. The chapter under consideration is a concise, if elementary, treatment of differential equations and not at all a list of isolated methods for solving the various types. The greater portion of the chapter is devoted to the consideration of ordinary differential equations and contains a section upon the calculus of variations, which is necessarily limited to the older theory and its classic problems. The part devoted to partial differential equations is pushed far enough to include the Ampere equation and its special case, the Monge equation, but with the frank acknowledgment that partial differential equations of the second order have not, as yet, been completely solved.

Illustrative examples are worked throughout the text but not much space is given to lists of problems to be solved by the student — a circumstance which would indicate, even to the casual reader, more meat than bone. The book is clearly and concisely written and lacks the heaviness which characterizes some German texts. It is furnished with a good index of subjects, as well as a register of names, and is singularly free from errors. Numerous historical notes and references are given, which enhance the value of the book to the careful student.

It is safe to state that the student who has mastered the contents of both volumes will be in a position to read technical literature with intelligence, and it is equally safe to say that no student can consider himself equipped to work in any corner of the mathematical field without an equivalent body of knowledge ready at hand, or without the spirit of careful inquiry which pervades Professor Czuber's two volumes.

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Traité de Mathématiques générales à l'Usage des Chimistes, Physiciens, Ingenieurs, et des Élèves des Facultés des Sciences. Par E. FABRY. Avec une preface de G. DARBOUX. Paris, Hermann, 1909, 8vo. 10 + 440 pages.

DURING the last few years the requirements for the "diplôme de licencié ès sciences" have undergone considerable change. The tendency has been to imitate somewhat the German custom and allow the student greater freedom in the choice of subjects taken for this diploma. Under the present system each faculty issues certificates for the work taken in that faculty and a student who has obtained three of these certificates, whether at the same session or at different sessions, has the right to a diploma. Previously there were only three modes of obtaining this diploma, but under the new regulations Darboux tells us that at Paris alone it can be obtained in 1771 different ways.

Among the certificates most frequently issued those of general mathematics are certainly included. This course in general mathematics forms a sort of transition between the mathematics of the Lycée and the advanced mathematics of the University. Such a course has always been necessary for the student of mechanics and physics and is fast becoming necessary for the student of chemistry. It was to meet this new demand for a course in general mathematics that Professor Fabry undertook the difficult task of writing a book that should meet the needs of the engineer, physicist, and chemist and at the same time serve as a preliminary training for those who expect to pursue the study of mathematics for its own sake. Such a book must combine rigor with simplicity and in this lies the difficulty of the task. It seems that the author has succeeded remarkably well in combining these two things. To be sure at times the subject becomes quite abstract, but the student of applied science can omit these parts without in the least marring the course for him.