Studien über die Methoden von C. Neumann und G. Robin zur Lösung der beiden Randwertaufgaben der Potentialtheorie. Von ERNST RICHARD NEUMANN. Leipzig, Teubner, 1905.

This work was submitted to the Jablonowski Gesellschaft of Leipzig in 1902 in response to the wish of the society "that the investigations of Poincaré's memoir entitled 'La méthode de Neumann et le problème de Dirichlet,' 1896, be in some particular essentially completed," and was awarded the prize offered. The author sets himself the task of establishing the convergence of the series obtained by C. Neumann and Robin for a region bounded by a surface which is not necessarily convex and for boundary values of which continuity alone is demanded. In doing so, however, his first object is complete rigor and uniformity of method of proof. The method employed is based upon the integrals used by Poincaré of the type

$$\int\left[\frac{\partial\phi}{\partial x}\frac{\partial\psi}{\partial x}+\frac{\partial\phi}{\partial y}\frac{\partial\psi}{\partial y}+\frac{\partial\phi}{\partial z}\frac{\partial\psi}{\partial z}\right]d\tau$$

applied to the successive potentials of C. Neumann and Robin and also to the "polar potentials" introduced by the author in a note in the *Göttinger Nachrichten,* 1899, page 291. These polar potentials are simply the successive Neumann and Robin potentials that would arise in forming the Green functions for the two corresponding problems, and their introduction goes far toward unifying and simplifying the discussion. Using them, together with his two theorems of the "constancy of the moment" (see pages 50–51, also *Mathematische Annalen,* volume 54, page 40), the author succeeds in establishing the limits which the terms, or more strictly in the case of the Neumann series, pairs of terms of his series approach (pages 112–114), but quite without establishing any criteria for the rapidity of approach necessary for a convergence proof. To attain this he finds it necessary to make use of a "Poincaré principle," namely that if U be the potential of any simple distribution on the boundary surface σ, the total mass being 0, then the ratio of the value of the integral

$$\int\left[\left(\frac{\partial U}{\partial x}\right)^2+\left(\frac{\partial U}{\partial y}\right)^2+\left(\frac{\partial U}{\partial z}\right)^2\right]d\tau$$

when extended over the space outside σ to its value when extended over the space within σ lies between two constants greater than 0. All the usual results with respect to the series follow immediately. But uniformity and rigor have been sacrificed, for it is explicitly stated that the question of the proof of the above theorem is left open. In Poincaré's work it is derived on the basis of certain "Poincaré transformations," without appeal to Dirichlet's principle, and E. R. Neumann's work deserves credit for crystallizing the difficulty of the situation in this one theorem, particularly in view of the fact that for a large class of surfaces it has been proven (Korn: Lehrbuch der Potentialtheorie, I, pages 241 and 245), and further generalization is probably possible.

The results desired by the Jablonowski Gesellschaft have on the other hand since been attained in a more far reaching manner through Fredholm's work, especially in its application by J. Plemelj (*Monatshefte für Mathematik und Physik*, volume 15, 1904). The matter of convergence is completely settled and interest in the work of the more immediate followers of Poincaré becomes historical in its nature, with the important exception of certain studies of the behavoir with respect to continuity of surface distributions and their derivatives, upon which the Fredholm method for the potential problems also depends. The present work is of value in both respects and is moreover to be commended for its style and arrangement.

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Vorlesungen über mathematische Näherungsmethoden. By Dr. OTTO BIERMANN. Braunschweig, Friedrich Vieweg und Sohn, 1905. 227 pp.

"In recent years much has been done to meet the requirements of those who find it necessary to make use of mathematical methods, and it is therefore surprising that as yet no book exists which treats of methods of approximation in mathematics in clear and concise form and so as not to require much preliminary mathematical knowledge." This, the first sentence of the preface, indicates clearly the object with which the book under review was written. We may say at once that it fulfills this object admirably.

The book is divided into six parts treating respectively of: I, calculation with exact and approximate numbers; II, numerical computation in higher analysis; III, the approximate