SHORTER NOTICES.

Arithmétique Graphique. Introduction a l'Etude des Fonctions Arithmétique. Par G. ARNOUX. Paris, Gauthier-Villars, 1906. xx + 225 pp.

THIS is the second of two volumes published by the author under the general title Arithmétique graphique. The first of these appeared in 1894 with the special title Les espaces arithmétiques hypermagiques. It has to do with magic squares, cubes, etc., but its methods are those of the theory of numbers. While it was still in press an abstract of it was presented to the Société mathématique de France by C. A. Laisant,* who was so impressed with its methods that he made the following prophecy: "Il y a en effet une telle originalité, une telle puissance d'invention dans les méthodes dont il s'agit, que je serais bien étonné si l'arithmétique n'arrivait pas quelque jour à les utiliser, soit pour obtenir des démonstrations plus simples de vérités connues déjà, soit pour découvrir des vérités nouvelles."

The starting point of Arnoux's method is the definition of a modular arithmetic space. A one-dimensional arithmetic space of modulus m consists of the m points of a straight line whose coordinates are $0, 1, 2, \cdots, m-1$. A two-dimensional arithmetic space consists of the m^2 points of a plane whose cartesian coordinates are the m^2 pairs of the integers $0, 1, 2, \cdots, m-1$. The extension to three or more dimensions is obvious. An arithmetic space of modulus m may be thought of as a *finite geometry* that corresponds to the *finite algebra* of integers modulo m. An arithmetic straight line in a two-dimensional arithmetic space consists of the m points whose coordinates satisfy a linear congruence of the form $ax + by + c \equiv 0 \pmod{m}$.† The points of such a line do not in general all lie on a euclidean straight line but there is a close connection between the two. For those points of a euclidean plane whose coordinates are integers may be arranged in an infinite number of arithmetic spaces of modulus m by reducing the coordinates modulo m. A euclidean straight line determined by two points A and B of one of these modular arithmetic spaces will

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† Arnoux gives an equivalent definition in terms of the function $ax + by$ and the modulus m.

cross an infinite number of other such arithmetic spaces, but it will contain only m points that have different pairs of coordinates. The points of the given arithmetic space that have these m different pairs of coordinates are the m points of the arithmetic straight line determined by A and B. An arithmetic plane in a modular arithmetic space of three dimensions consists of the $m(m+1)$ points whose coordinates satisfy a congruence of the form $ax + by + cz + d \equiv 0 \pmod{m}$. An arithmetic line in such a space consists of the points common to two arithmetic planes.

The most interesting arithmetic spaces are those in which the modulus is a prime number p, for in the finite algebra of integers modulo p the formal laws of addition, subtraction, multiplication and division are as in ordinary algebra. Arnoux's use of these finite geometries for the construction of magic squares is quite original and well worth reading.

The book under review is an attempt to illuminate the theory of arithmetic functions by the use of modular arithmetic spaces which furnish a means of graphical representation and make it possible to use the convenient language of geometry, but in the reviewer's opinion it fails to fulfill the prophecy of Laisant quoted above.

The book is very well described by its author in the first sentence of the preface in these words: "Ce livre n'est point un traité didactique sur les fonctions arithmétiques, mais plutôt une simple causerie sur ce sujet si intéressant et si peu connu." This precludes many criticisms that might otherwise be made, for it is not a good book for a beginner in the subject although it is described in the title as an introduction to the theory of arithmetic functions. The method throughout is largely synthetic and inductive. The many tables and the special cases used in leading up to the proofs of theorems make the number of pages large for the subject matter treated. The use of modular arithmetic spaces is interesting but not so effective as it was in the author's earlier work on magic squares in which these finite geometries played such an important part.

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