21. Professor Pupin's paper discusses the quasi-discontinuous variations of the electric current in a sectional conductor when a steady electromotive force is impressed upon it. Thomson's mathematical discussion of the electric cable problem in 1855 and Kirchhoff's solution in 1858 of Thomson's problem in more general form are special cases of that here treated. The paper contributes an addition to the exceedingly small number of existing illustrations of the application of the Lagrangian method of solving a certain class of differential equations by a finite series of harmonics, the extension of which led to the discovery of Fourier's series. The solution discussed in the paper seems to offer a field for physical applications.

22. Dr. Quinn presented two preliminary theorems on properties of the cissoid, which suggest a mode of constructing a linkage for its kinematic description.

F. N. COLE,
Secretary.

NOTE ON CERTAIN GROUPS OF TRANSFORMATIONS OF THE PLANE INTO ITSELF.

BY DR. PETER FIELD.

(Read before the American Mathematical Society, December 29, 1905.)

IN the study of a plane curve or other configuration determined by five points,* four of the points may be taken as fixed and all the curves can be obtained by taking the fifth as any point in a given one of the one hundred and twenty regions determined in Professor Slaught's thesis.† Professor Slaught's diagram applies to the case of five real points. It is the purpose of this note to indicate diagrams corresponding to the case when four or two of the points are imaginary.

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* Del Pezzo, *Rendiconti Accad. scienze fisiche matem.*, Napoli, ser. 2, vol. 3 (1889), pp. 46–49. Field, *Transactions Amer. Math., Society*, January, 1906.

† "The cross ratio group of one hundred and twenty quadratic Cremona transformations of the plane," *Amer. Journ. of Math.*, vols. 22 and 23. Also see Moore, "The cross ratio group of n! Cremona transformations of order n—3 in flat space of n—3 dimensions," *Amer. Journ. of Math.*, vol. 22, No. 3, pp. 279-291, and Kantor, Theorie der endlichen Gruppen von eindeutigen Transformationen in der Ebene (Berlin, 1895).

1. *Four Imaginary Points.*

Let the points be $a \equiv (i: 0: 1)$, $b \equiv (-i: 0: 1$, $c \equiv (0: i: 1)$, $d \equiv (0: -i: 1)$, the fifth point being any real point $x_1: y_1: z_1$. It is clear that the fifth point can not be projected into either of the first four, while the different ways in which the first four can be projected into themselves are given by the transitive group of order 8 and degree 4 [the generators of the group might be taken as the substitutions $(adbc)$ and (ab)]. If $z=0$ be the line at infinity, the fifth point may be taken as any point in one of the eight regions bounded by the lines $x=0$, $y=0$, $x-y=0$, $x+y=0$. This is evident when it is noticed that the generators of the group of transformations corresponding to the preceding substitution are $x': y': z' = y_1: -x_1: z_1$ and $x': y': z' = -x_1: y_1: z_1$.

2. *Two Imaginary Points.*

In case but two of the points are imaginary let the five points be taken as follows: $g_1 \equiv (0:0:1)$, $h_1 \equiv (1:1:1)$, $c \equiv (i:1:0)$, $d \equiv (-i:1:0)$, $e \equiv (x_1:y_1:z_1.)$ Four of the preceding five points can be projected into the first four in twelve different ways, because any one of the three real points may be projected into g_1 and either of the two remaining into h_1, while c and d may remain invariant or be interchanged. To project (g_1, h_1, d, c), (g_1, e_1, c, d), (h_1, g_1, c, d) into (g_1, h_1, c, d) we have the transformations

$$x': y': z' = y: x: z, \qquad x': y': z' = (x_1 + y_1)x - (x_1 - y_1)y:$$

$$(x_1 - y_1)x + (x_1 + y_1)y: \frac{x_1^2 + y_1^2}{z_1} z, \quad x': y': z' = z - x: z - y: z.$$

For these three cases the coördinates of the new fifth point in terms of x_1, y_1, z_1, are respectively

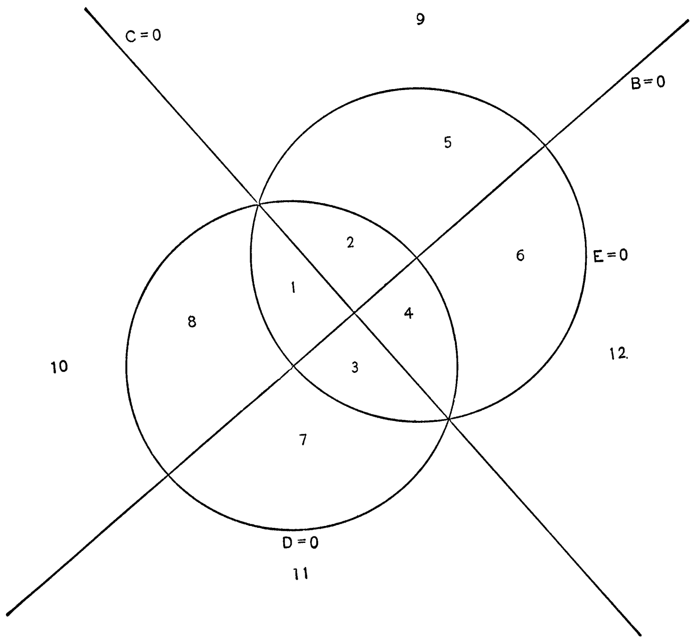
$$x': y': z' = y_1: x_1: z_1, \tag{1}$$

$$x': y': z' = 2y_1z_1: 2x_1z_1: x_1^2 + y_1^2, \tag{2}$$

$$x': y': z' = z_1 - x_1: z_1 - y_1: z_1. \tag{3}$$

These last transformations generate a group of order 12. The boundaries of the regions are the curves $A \equiv z = 0$,

$B \equiv x - y = 0$, $C \equiv x + y - z = 0$, $D \equiv x^2 + y^2 - 2z^2 = 0$, $E \equiv (x - z)^2 + (y - z)^2 - 2z^2 = 0$. The way in which the



regions are permuted under the transformations (1), (2), (3) is shown in the following table:

(1)	(13), (24), (56), (78), (9 12), (10 11),
(2)	(1 12), (26), (39), (45), (7 10), (8 11),
(3)	(14), (23), (57), (68), (9 11), (10 12),

In the figure above the line A is taken as the line at infinity.

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