pal normal, binormal, curvature, torsion, osculating circle, osculating helix, in the case of twisted curves; and in the case of surfaces we find the definitions and more important properties of principal radii of curvature, mean and total curvature, ruled, developable, and minimal surfaces, lines of curvature, etc., subjects, many of them, that an American student of engineering never hears anything about. The book also contains a large number of formulas, which are made easily accessible by means of a good index.

The treatment of some of the subjects could easily be criticised on the score of rigor and some of the propositions appear to be stated with too much generality; but the niceties of modern rigor must not be insisted upon in such an elementary and one might almost say popular exposition, and the inaccuracies we have noted may well be pardoned in view of the general excellence of the whole. The typographical errors that we have noticed are few, and all of such an evident character, that it is quite unnecessary to enumerate them. We believe that M. Pionchon is to be congratulated on writiug a thoroughly serviceable and very readable book.

J. W. YOUNG.

Étude sur les Quantitées mathématiques. Grandeurs dirigées. Quaternions. CLARO CORNELIO DASSEN. Paris, A. Hermann, 1903. vi + 133 pp.

M. DASSEN tells us in the introduction that it has been his object in writing this book, to "clear up and popularize the notions which lie at the foundations of pure mathematics." The author takes the extreme utilitarian view, and will admit into the science no "play of definitions and symbols" which cannot be put to "some practical use," lest he be beguiled into a realm of mere cabalistic hocus-pocus. He admits, however, that some intrinsically useless investigations may have a certain indirect value, and then gives us a hint on the breadth of his mathematical learning as follows: "The so-called non-euclidean geometry, for example, though useless in itself, because it does not correspond to experience, has nevertheless shown itself indirectly useful in proving that the euclidean geometry, the only one that does correspond to experience, is not apodictically true and has hence served to refute the arguments of Kant on the a-priority of the concept of space" (page 2).

With these introductory sentiments the author takes up his work of clearing up the notions that form the basis of algebra. A mathematical quantity is defined (page 3) as "anything that can be thought of as consisting of an aggregate of parts or 'units' such that they may be subjected to combinations called 'operations' either among themselves or with analogous elements of other aggregates having the same unit." Subtraction (page 14) is pronounced impossible when the minuend is less than the subtrahend, and (page 17) the symbol $-a$ is declared meaningless and incapable of interpretation, unless to the notion of quantity be added that of direction (the discussion of which is reserved for the later chapters). Yet a few pages further on (page 20), we are told that "in order to preserve the rules of calculation," it is necessary to give a certain meaning to the symbols a^0 and a^{-n}, and are referred to the great advantage resulting from the use of this notation, with never a word in justification of operating with the "meaningless" symbol $-n$.

We agreed with the author (page 17), when he showed that the symbol $0/0$ could represent any quantity; but find some difficulty in realizing (page 20) that "0^0 evidently signifies zero," when the symbol a^0 has just been defined by the relation $a^m/a^m = a^{m-m}$. But the author certainly means what he says, for later (page 25) we find: "The expression $\log_0 0$ equals zero (for $0^0 = 0$)." It seems to the reviewer that the author in pursuance of his avowed object should have elucidated this point a little more fully.

The author probably reaches the highest point in his progress toward the simplification of mathematics in his proof of the existence of $\sqrt[b]{a}$. He observes (page 26) that all continuous mathematical quantities can be represented by lengths, and then says: "Now it is easy, by graphical constructions, to determine lengths connected with others by the relation $c^b = a$, where b is any whole number and a any length whatever, whence $\cdots c = \sqrt[b]{a}$, a result which shows that even when $\sqrt[b]{a}$ is meaningless in itself it corresponds nevertheless to a length." He then describes "for example" the necessary construction for $b = 2$. We wonder if M. Dassen could not have given a more complete treatment of this point. We are not in the habit of regarding constructions requiring complicated linkages or recourse to hyperspace as "easy." That the author regards the above as a valid proof is clear from the fact that later (page 28) he speaks of "the quantity $\sqrt[b]{a}$, of which we have above shown the existence." We are told further (page 28) that in case of continuous magnitude the presence of quantities other than the rational is necessarily imposed by the assumption of the infinite divisibility of such magnitude, "for otherwise the division into parts would be necessarily limited." This sounds very clear and simple; and yet we are troubled by the fact that infinite divisibility in M. Dassen's sense seems to us to imply not necessarily a continuum, but only an aggregate everywhere dense in itself, for the representation of which the rational numbers suffice.

Enough has probably been said to indicate the character of M. Dassen's philosophy and the mathematical knowledge on which it is based, so that we may refer anyone interested in its further development to the remaining two chapters of the book itself, which have to do with directed quantities in the plane and in space respectively. In an appended note the author pays his respects to the work of Tannery and Kronecker regarding the founding of analysis on the concept of the positive integer alone, and pronounces it quite useless and a mere jugglery of symbols, "at which one is justly shocked." From what precedes, the fact that he has entirely missed the real object of such work is not surprising.

Before closing, we would, however, refer to a feature of the work which is of considerable interest. The author has scattered through the text a very large number of historical data. These are quite independent of his philosophy and seem to be drawn from reliable and often not easily accessible sources.

J. W. YOUNG.

A Treatise on Differential Equations. By ANDREW RUSSELL FORSYTH. Third Edition. Macmillan and Co., 1903.

IT is not an easy matter to review a book which, like the present one, has been before the public so many years, the first edition having appeared in 1885 and the second in 1888. That this treatise has many virtues has been quite conclusively shown by its success. In fact in English-speaking countries the domain of differential equations has, since 1885, been synonymous with the name of Mr. Forsyth, at least in the minds of that great category of students whose knowledge comes from text-books only.