spherical surfaces; for the sake of brevity they are referred to as A-surfaces. In the present paper he discusses the triply-orthogonal systems in which one family of surfaces are of this kind and calls such a system an A-system. With each A-surface of such a system there is *associated* a pseudospherical surface, namely, the one with the given representation of its lines of curvature. It is shown that all of these associated surfaces form part of another triply orthogonal system; hence all the A-systems are obtained from pseudospherical systems by transformations of Combescure and Darboux. In the former paper transformations of A-surfaces were discovered which are generalizations of the transformations of Bianchi and Bäcklund for pseudospherical surfaces. The determination is made of the transformations changing all the A-surfaces of an A-system into other A-surfaces forming a family of a new A-system. The following particular A-systems are considered: 1), when the associated pseudospherical surfaces form part of a system of Weingarten; 2), when the orthogonal trajectories of the A-surfaces are plane, or the lines of curvature in one system upon these surfaces are plane; 3), when the orthogonal trajectories of the A-surfaces have constant curvatures; 4), when the lines of curvature in one system are spherical, or the transformed surface, under right angle, have this property; 5), when one of the other families of surfaces of an A-system contains only spheres; and 6), when the A-surfaces of an A-system are surfaces of Bianchi of the elliptic, parabolic or hyperbolic types.

F. N. Cole,
Secretary.

THE SEPTEMBER MEETING OF THE SAN FRANCISCO SECTION.

The eighth regular meeting of the San Francisco Section of the American Mathematical Society was held at the University of California, on Saturday, September 30, 1905. The following seventeen members were present:

Professor R. E. Allardice, Professor G. C. Edwards, Professor M. W. Haskell, Professor D. N. Lehmer, Professor A. O. Leuschner, Dr. J. H. McDonald, Dr. W. A. Manning, Professor G. A. Miller, Professor H. C. Moreno, Dr. B. L.

Newkirk, Professor C. A. Noble, Dr. T. M. Putnam, Mr. Arthur Ranum, Professor Irving Stringham, Mr. L. C. Walker, Professor A. W. Whitney, Professor E. J. Wilczynski.

Professor M. W. Haskell presided during the two sessions. The following officers were elected for the ensuing year: Professor R. E. Allardice, chairman; Professor G. A. Miller, secretary; Professors E. J. Wilczynski, D. N. Lehmer, G. A. Miller, programme committee.

The following papers were read at this meeting:

(1) Professor C. A. NOBLE: "Note on loxodromes."

(2) Dr. W. A. MANNING: "Groups in which more than half of the operators may correspond to their inverses."

(3) Professor M. W. HASKELL: "A new canonical form of the binary sextic."

(4) Professor A. O. LEUSCHNER: "On a new method of determining orbits."

(5) Mr. ARTHUR RANUM: "The representation of linear fractional congruence groups with a composite modulus as permutation groups."

(6) Professor E. J. WILCZYNSKI: "On a system of partial differential equations in involution."

(7) Professor G. A. MILLER: "The groups which contain only three operators which are squares."

(8) Professor R. E. MORITZ: "On logarithmic involution, the commutative arithmetic process of the third order."

(9) Professor L. E. DICKSON: "The abstract group simply isomorphic with the general linear group in an arbitrary field."

(10) Professor L. E. DICKSON: "The abstract group simply isomorphic with the symmetric group."

(11) Professor M. W. HASKELL: "On a class of covariants of order n which give rise to a birational transformation."

In the absence of the authors the papers by Professors Moritz and Dickson were read by title. Abstracts of all the papers are given below. The abstracts are numbered to correspond to the titles in the list above.

1. If the sequence of meridians and of parallels of latitude on a surface of revolution be so determined as to cover the surface with a network of similar rectangles, the diagonals of these rectangles will be loxodromes. The determination of the appropriate sequence involves a simple differential equation, the solution of which leads, by quadrature, to the finite equation

of the ∞^2 loxodromes on the surface. Professor Noble's paper is a contribution to the methods of determining these loxodromes but does not claim any new results.

2. Professor Miller has called attention to the fact that a group is abelian if more than three fourths of its operators may be made to correspond to their inverses. Dr. Manning shows that if just three fourths or just two thirds the operators of a group may correspond to their inverses, certain groups of comparatively simple composition are determined. If more than two thirds, then three fourths or all the operators of the group may be made to correspond to their inverses.

3. It is well known that the binary sextic can be expressed as the sum of four sixth powers in an infinity of ways, but not much use has been made of such a canonical form owing to its lack of definiteness. Professor Haskell shows that an assumed relation between the four linear forms involved gives rise to a number of useful canonical forms. For instance, the reduction is possible in two ways if the linear forms in question be equianharmonic, in three ways if they be harmonic, in one way if a certain simultaneous covariant of their product and of the given binary sextic vanishes. In this last case the product is a covariant of order four and degree five.

4. Professor Leuschner's paper was very closely related to the one presented at the last meeting of this Section. It indicated some further applications of the "short method" and gave instances of the amount of time that could be saved by means of it. The application to the determination of the orbit of the seventh satellite of Jupiter was given in some detail.

5. The congruence group of linear fractional substitutions in one variable, whose modulus m is composite, is represented by Mr. Ranum as a transitive permutation group on the totality of symbols of the form $[a, b]$, or a/b, such that the greatest common divisor of a, b, and m is 1, and the symbols $[a, b]$ and $[ka, kb]$ are identical. If

$$m = \prod_{i=1}^{i=n} p_i^{a_i},$$

then the number of symbols is

$$m\prod_{i=1}^{i=n}\left(1+\frac{1}{p_i}\right).$$

Every separation of m into factors r and s prime to each other gives rise to two sets of imprimitive systems of the group, viz., the systems of symbols which are congruent to one another, mod r and mod s, respectively. Moreover, if m contains a power of a prime higher than the first ($m = p^\alpha r$), then there are sets of imprimitive systems of symbols which are at the same time congruent to one another, mod r and mod p^k, for every value of k from 1 to $\alpha - 1$. Corresponding to every set of systems of imprimitivity there is an invariant subgroup for which they are sets of intransitivity.

6. In general the simultaneous system of partial differential equations

$$(1)\qquad \begin{aligned} \frac{\partial^2 z}{\partial x^2} &= a\frac{\partial^2 z}{\partial x \partial y} + b\frac{\partial z}{\partial x} + c\frac{\partial z}{\partial y} + dz, \\ \frac{\partial^2 z}{\partial y^2} &= a'\frac{\partial^2 z}{\partial x \partial y} + b'\frac{\partial z}{\partial x} + c'\frac{\partial z}{\partial y} + d'z \end{aligned}$$

has four linearly independent solutions, say $z_1, \cdots, z_4$. Upon this fact may be based a projective differential geometry of surfaces. If however, certain relations are satisfied between the coefficients, the common solutions of the two equations may depend upon an arbitrary function. This is the case in which the two equations are said to be in involution. Professor Wilczynski examines this exceptional case and finds that it corresponds to the case of developable surfaces. The projective differential geometry of all non-developable surfaces may therefore be based upon the invariant theory of system (1).

7. Every group besides the abelian group of order 2^α and of type $(1, 1, 1, \cdots)$ contains at least two operators which are squares of other operators in the group. If there are only two such operators they constitute an invariant subgroup and the corresponding quotient group contains only one operator which is a square. All the groups which satisfy this condition have been determined. Professor Miller's paper is devoted to a complete determination of the groups involving just three operators which are squares. The main results may be stated as follows: All these groups are direct products of the abelian

group of order 2^γ and of type $(1, 1, 1, \cdots)$ into the cyclic group of order 3, the symmetric group of order 6, or a group of order 2^β. When $\beta = 4$, there are two such groups of order 2^β. When $\beta = 5$, there are three; when $\beta > 5$ there are four. In the last case, two of the groups have a commutator subgroup of order two while the other two have a commutator subgroup of order four. Each of these four groups has exactly $2^{\beta-2} - 1$ subgroups of order $2^{\beta-1}$. The paper has been offered to the *Transactions* for publication.

8. Eisenstein, Woepcke, Paugger, Gerlach and De Morgan have attempted to set up manageable processes of higher orders than the third by following analogies of form or structure of the ordinary processes. All these attempts were unsuccessful, except De Morgan's, whose definition seems however arbitrary and artificial. Professor Moritz points out that simplicity of form, or unity in structure, however desirable, are less essential than unity in fundamental laws. The principle of permanence, which has governed the extension of the number body, should be recognized in the extension of the number processes. This principle demands that ordinary involution, a^b, be replaced by the process $a^{\log b}$ which may be appropriately termed logarithmic involution. From this in turn the principle of permanence leads directly to the higher processes of De Morgan. Logarithmic involution obeys all the laws of the two lower processes and could be made to serve the purposes of ordinary involution. The paper is accompanied by two tables, one for performing logarithmic involutions directly, the other by means of logarithms. It will be offered to the *Annals of Mathematics* for publication.

9. It is shown in the paper of Professor Dickson that the group of all n-ary linear homogeneous substitutions of determinant unity in any given field F is simply isomorphic with the abstract group generated by the operators $B_{ij\lambda}(i, j = 1, \cdots, n;\ i \neq j;\ \lambda$ ranging over $F)$, subject to the generational relations $B_{ij\lambda}B_{ij\mu} = B_{ij\lambda+\mu}$, $B_{ij\lambda}$ commutative with $B_{kl\mu}$, $B_{ik\mu}$, $B_{kj\mu}$

$$B_{ij\lambda}^{-1}B_{jk\mu}^{-1}B_{ij\lambda}B_{jk\mu} = B_{ik-\lambda\mu}, \quad B_{ij\lambda}B_{ji\mu}B_{ij\nu} = B_{ji\mu\nu k^{-1}}B_{ijk}B_{ji\lambda\mu k^{-1}}$$

for $k \equiv \lambda + \nu + \lambda\mu\nu \neq 0$, where i, j, k, l, are any distinct integers $\leqq n$ while λ, μ, ν are any marks of F subject to a condition only in the last set of relations. In the concrete group,

$B_{ij\lambda}$ represents the substitution which alters only ξ_i, replacing it by $\xi_i + \lambda\xi_j$.

Another symmetric definition employs the generators $B_{ii+1\lambda}$ and $B_{i+1i\lambda}$ ($i = 1, 2, \cdots, n - 1$ variables).

As corollaries are deduced symmetric definitions of the abstract groups isomorphic with the linear fractional and linear non-homogeneous groups on $n - 1$ variables.

10. In his second paper Professor Dickson points out that the symmetric group on n letters is simply isomorphic with the abstract group generated by the operators $T_{ij} \equiv T_{ji}$ ($i, j = 1, \cdots, n$; $i \neq j$) subject to the relations $T_{ij}^2 = I$, $T_{rk}^{-1} T_{rs} T_{rk} = T_{sk}$, T_{ij} commutative with T_{kl} when i, j, k, l, are all distinct. The proof is immediate. Another symmetric definition by fewer generators is due to Professor Moore, *Proceedings of the London Mathematical Society*, volume 28, page 357.

11. Professor Haskell has shown before that there is a birational transformation between the coëfficients of a binary cubic and of its cubicovariant. In seeking for a generalization of this relation he finds that the binary n-ic, if $n = 4m - 1$, possesses a covariant of order n whose coefficients are then related to those of the ground form; if $n = 4m$, there is such a relation provided a certain invariant vanishes; while for $n = 4m + 1$ or $n = 4m + 2$, there is no covariant bearing this property.

The next meeting of the Section will be held at Stanford University on February 24, 1906. The newly elected officers will begin their term of office with this meeting.

G. A. MILLER,
Secretary of the Section.

NOTE ON LOXODROMES.

BY PROFESSOR C. A. NOBLE.

(Read before the San Francisco Section of the American Mathematical Society, September 30, 1905.)

A LOXODROME is a curve on a surface of revolution which meets the meridians at a constant angle. If the meridians and the parallels of latitude on such a surface can be so selected as to constitute a network of similar infinitesimal rectangles, the diagonals of these rectangles will give loxodromes.