THE OCTOBER MEETING OF THE AMERICAN MATHEMATICAL SOCIETY.

THE one hundred and twenty-fifth regular meeting of the AMERICAN MATHEMATICAL SOCIETY was held in New York City on Saturday, October 28, 1905. The attendance at the two sessions included the following thirty members of the Society:

Professor D. P. Bartlett, Professor G. A. Bliss, Dr. W. H. Bussey, Professor F. N. Cole, Miss E. B. Cowley, Dr. W. S. Dennett, Professor L. P. Eisenhart, Professor T. S. Fiske, Mr. S. A. Joffe, Dr. Edward Kasner, Professor C. J. Keyser, Dr. G. H. Ling, Professor E. O. Lovett, Professor James Maclay, Dr. Max Mason, Professor H. B. Mitchell, Dr. C. L. E. Moore, Professor W. F. Osgood, Professor James Pierpont, Miss I. M. Schottenfels, Professor D. E. Smith, Professor P. F. Smith, Dr. C. E. Stromquist, Professor J. H. Tanner, Professor H. D. Thompson, Miss Mary Underhill, Professor J. M. Van Vleck, Professor Oswald Veblen, Professor J. W. Young, Professor H. S. White.

The simultaneous meeting of the American Physical Society afforded opportunity for the cultivation of cordial relations. The luncheon in the interval between the sessions and an informal dinner in the evening was arranged to bring the members of the two Societies into closer communion.

At the meeting of the Mathematical Society President W. F. Osgood occupied the chair. The Council announced the election of the following persons to membership in the Society: Professor O. P. Akers, Allegheny College; Dr. R. B. Allen, Clark University; Professor Ernesto Cesàro, University of Naples; Lieutenant Colonel A. J. C. Cunningham, London, Eng.; Miss M. E. Decherd, University of Texas; Mr. W. W. Hart, Shortridge High School, Indianapolis, Ind.; Mr. H. N. Olsen, Bethany College; Mr. F. H. Smith, Southwestern Christian College. Twelve applications for membership in the Society were received.

A list of nominations for officers and other members of the Council was adopted and ordered placed on the official ballot for the annual election at the December meeting. Dr. W. H. Bussey was appointed assistant secretary of the Society.

The President was authorized to appoint a committee to arrange for the summer meeting and colloquium for 1906. The members of this committee are Professors Maschke, Pierpont, P. F. Smith, H. S. White, and the Secretary. A committee was also appointed to audit the Treasurer's accounts for the current year.

The following papers were read at this meeting:

(1) Professor W. B. CARVER: "On the Cayley-Veronese class of configurations."

(2) Professor JAMES PIERPONT: "Multiple improper integrals."

(3) Dr. EDWARD KASNER: "On the geodesics passing through a given point of a surface."

(4) Professor H. S. WHITE: "Poncelet quadrilaterals on a curve of the third order and a conic."

(5) Dr. MAX MASON and Professor G. A. BLISS: "A problem of the calculus of variations in which the integrand function is discontinuous."

(6) Professor G. A. MILLER: "Groups generated by two operators which transform each other into the same power."

(7) Dr. BURKE SMITH: "Determination of associated surfaces."

(8) Professor L. P. EISENHART: "Certain triply orthogonal systems of surfaces."

Professor Carver's paper was presented to the Society through Professor T. S. Fiske. This paper was read by title as were also the papers of Professor Miller, Dr. Smith, and Professor Eisenhart. Professor Carver's paper was published in the October *Transactions*. Abstracts of the remaining papers follow below. The abstracts are numbered to correspond to the titles in the list above.

2. Professor Pierpont's paper was a continuation of his paper read at the April meeting of the Society. In that paper he developed the theory of outer multiple proper integrals. In the present paper the cases that either the integrand or field of integration become infinite are treated. The notions of cell and of division of space into cells developed in the former paper play a fundamental role in the present paper. In addition, the notions of uniform and regular evanescence of the singular integral, here introduced for the first time, enable the author to develop the theory of improper integrals with great simplicity.

3. If the geodesics passing through a given point O of a sur-

face are projected orthogonally upon the tangent plane at that point, a system of plane curves is obtained with, in general, inflexions at O. The only cases where the projected curve has third order contact with its tangent line arise (1) when the geodesic starts from O in a principal or asymptotic direction, or (2) when O is an umbilic. In the latter case there are three exceptional curves in the system having fourth order contact with their tangent lines. Dr. Kasner next considers the projection of the geodesics passing through a point O' in the neighborhood of O; this leads to an interesting correspondence analogous to that of conjugate directions. Incidentally, a number of geometric interpretations of quantities of the third order in the theory of surfaces are brought to light.

4. Under certain conditions a triangle can vary freely with its vertices sliding upon a plane cubic curve and its sides continually touching a curve of the second class. Professor White shows that in this case necessarily a quadrilateral varies in the same way, with its six vertices on the cubic. The analytic conditions, three in number, are found by means of results contained in his paper on "Twisted cubic curves that have a directrix" (*Transactions*, volume 4 (1903)).

5. In a medium in which the index of refraction is variable, a ray of light usually follows the curve on which the time of passage is a minimum. The time is expressible as a definite integral, so that the problem of the determination of the path is one belonging to the calculus of variations. If the ray passes from one medium to another the path will have an angle at the dividing surface, where the index of refraction is discontinuous. This suggests at once the subject of the paper of Dr. Mason and Professor Bliss: the study of the problem of the calculus of variations in the plane when the function under the integral sign is discontinuous along a given curve. Some of the well-known results with regard to necessary conditions apply at once. The more important parts of the paper are the discussions of the condition corresponding to Jacobi's in the usual problem, the construction of a field, and the derivation of *sufficient* conditions for the integral to be a minimum.

6. Let t_1, t_2 be any two operators which transform each other into the α power. Since their commutator, $t_1^{-1}t_2^{-1}t_1t_2=t_1^{\alpha-1}=t_2^{\alpha-1}$,

is commutative with each of these operators, the group G generated by t_1, t_2 is the direct product of its Sylow subgroups. From the same relations it follows that the orders of t_1, t_2 are divisors of $(\alpha - 1)^2$. In particular, two distinct operators cannot transform each other into their squares but they may be so chosen as to transform each other into any other power which is not zero. Professor Miller first considers the case when $\alpha - 1$ is a prime number and finds that there are just one abelian and one non-abelian group which are generated by such operators. In particular, the quaternion group is the only possible non-abelian group which is generated by two operators which transform each other into their third powers. This group may also be defined by the fact that it is generated by two non-commutative operators which transform each other into their inverses.

When $\alpha - 1 = p^m$, p being a prime, there are $\frac{1}{2}m(m+1)$ non-abelian groups which are generated by two operators which transform each other into the α power. The orders of the two generators must be equal to each other and all of these groups are conformal with abelian groups except when $p = 1$ and $m = 1$. In this special case we arrive at the quaternion group, as noted above. As G is the direct product of its Sylow subgroups the case where $\alpha - 1$ is not a power of a prime is readily deduced from this case. Since the abelian Sylow subgroups present no difficulties, the author has confined his attention to the case where all the Sylow subgroups are non-abelian. In this case the two generators t_1, t_2 are of the same order, and the number of possible groups which may be generated by t_1, t_2 has been determined.

7. Dr. Smith develops a method for finding the associate of a given surface when it is referred to its asymptotic lines. The formulas developed enable one to write down the cartesian coordinates of the associated surfaces at once, after one has solved an equation of the Laplace type. It is also shown how one can obtain the equations of pairs of associated surfaces by solving an equation of the Laplace type and performing six quadratures.

8. In the April number of the *American Journal of Mathematics* Professor Eisenhart discussed surfaces with the same spherical representation of their lines of curvature as pseudo-

spherical surfaces; for the sake of brevity they are referred to as A-surfaces. In the present paper he discusses the triply-orthogonal systems in which one family of surfaces are of this kind and calls such a system an A-system. With each A-surface of such a system there is *associated* a pseudospherical surface, namely, the one with the given representation of its lines of curvature. It is shown that all of these associated surfaces form part of another triply orthogonal system; hence all the A-systems are obtained from pseudospherical systems by transformations of Combescure and Darboux. In the former paper transformations of A-surfaces were discovered which are generalizations of the transformations of Bianchi and Bäcklund for pseudospherical surfaces. The determination is made of the transformations changing all the A-surfaces of an A-system into other A-surfaces forming a family of a new A-system. The following particular A-systems are considered: 1), when the associated pseudospherical surfaces form part of a system of Weingarten; 2), when the orthogonal trajectories of the A-surfaces are plane, or the lines of curvature in one system upon these surfaces are plane; 3), when the orthogonal trajectories of the A-surfaces have constant curvatures; 4), when the lines of curvature in one system are spherical, or the transformed surface, under right angle, have this property; 5), when one of the other families of surfaces of an A-system contains only spheres; and 6), when the A-surfaces of an A-system are surfaces of Bianchi of the elliptic, parabolic or hyperbolic types.

F. N. COLE,
Secretary.

THE SEPTEMBER MEETING OF THE SAN FRANCISCO SECTION.

THE eighth regular meeting of the San Francisco Section of the AMERICAN MATHEMATICAL SOCIETY was held at the University of California, on Saturday, September 30, 1905. The following seventeen members were present:

Professor R. E. Allardice, Professor G. C. Edwards, Professor M. W. Haskell, Professor D. N. Lehmer, Professor A. O. Leuschner, Dr. J. H. McDonald, Dr. W. A. Manning, Professor G. A. Miller, Professor H. C. Moreno, Dr. B. L.