THE ELEVENTH ANNUAL MEETING OF THE AMERICAN MATHEMATICAL SOCIETY.

THE Eleventh Annual Meeting of the AMERICAN MATHEMATICAL SOCIETY was held in New York City on Thursday and Friday, December 29–30, 1904. The attendance included the following forty-nine members of the Society:

Mr. Joseph Allen, Dr. Grace Andrews, Professor Maxime Bôcher, Professor E. W. Brown, Dr. A. B. Coble, Professor F. N. Cole, Miss E. B. Cowley, Dr. D. R. Curtiss, Dr. W. S. Dennett, Mr. E. P. R. Duval, Dr. William Findlay, Professor T. S. Fiske, Dr. A. S. Gale, Miss Alice B. Gould, Professor J. G. Hardy, Professor James Harkness, Professor H. E. Hawkes, Professor L. I. Hewes, Dr. E. V. Huntington, Mr. S. A. Joffe, Dr. Edward Kasner, Professor C. J. Keyser, Professor P. A. Lambert, Mr. L. L. Locke, Professor E. O. Lovett, Dr. Max Mason, Professor W. H. Metzler, Mr. C. L. E. Moore, Professor E. H. Moore, Professor Frank Morley, Professor G. D. Olds, Miss Amy Rayson, Professor C. G. Rockwood, Professor E. D. Roe, Dr. Arthur Schultze, Mr. C. H. Sisam, Professor Virgil Snyder, Professor W. B. Smith, Dr. E. L. Stabler, Dr. C. E. Stromquist, President T. H. Taliaferro, Professor J. H. Tanner, Professor H. D. Thompson, Professor H. W. Tyler, Professor J. H. Van Amringe, Professor Anna L. Van Benschoten, Professor E. B. Van Vleck, Dr. E. B. Wilson, Mr. J. E. Wright.

A morning and an afternoon session were held on Thursday; a morning session on Friday sufficed to complete the work of the meeting. The retiring President of the Society, Professor Thomas S. Fiske, occupied the chair, being relieved by Professor G. D. Olds and Vice-President elect Professor E. W. Brown. At the opening of the afternoon session on Thursday, Professor Fiske delivered his presidential address on "Mathematical progress in America."

The Council announced the election of the following persons to membership in the society: Mr. G. I. Gavett, Stanford University; Mr. M. E. Graber, Heidelberg University, Tiffin, Ohio; Mr. E. B. Lytle, University of Illinois; Professor R. E. Moritz, University of Washington; Dr. B. L. Newkirk, University of California. Fourteen applications for membership in the Society were received.

A committee was appointed to arrange for the summer meeting and to consider the question of holding a colloquium in connection with that meeting.

Reports were presented by the Treasurer, Librarian, and Auditing Committee. These reports have appeared in the Annual Register. The membership of the Society has increased during the past year from 455 to 473. The number of papers presented at the meetings was 118; the total attendance of members at all meetings was 280; 160 members attended at least one meeting during the year. The library now contains about 2000 volumes; a complete catalogue is included in the Annual Register. The Treasurer's report shows a balance of $3884.28 on hand December 27, 1904; of this balance $1723.07 is credited to the life-membership fund.

An informal dinner on Thursday evening, attended by about thirty-five of the members, added much to the pleasure of the meeting.

At the annual election, which closed on Friday morning, the following officers and other members of the Council were chosen:

President,	Professor W. F. OSGOOD.
Vice-Presidents,	Professor E. W. BROWN,
	Professor JAMES PIERPONT.
Secretary,	Professor F. N. COLE.
Treasurer,	Dr. W. S. DENNETT.
Librarian,	Professor D. E. SMITH.

Committee of Publication,

Professor F. N. COLE,
Professor ALEXANDER ZIWET,
Professor D. E. SMITH.

Members of Council to serve until December, 1907,

Professor E. R. HEDRICK,	Professor E. O. LOVETT,
Professor T. F. HOLGATE,	Professor L. A. WAIT.

The following papers were read at the annual meeting:

(1) Dr. MAX MASON: "The doubly periodic solutions of Poisson's equation in the plane."

(2) Professor VIRGIL SNYDER: "On the forms of sextic scrolls having no rectilinear directrix."

(3) Dr. A. B. COBLE: "Some applications of a theorem in the theory of forms."

(4) Professor L. E. DICKSON : "The group of a tactical configuration."

(5) Professor T. S. FISKE : Presidential address, "Mathematical progress in America."

(6) M. MAURICE FRÉCHET : "Sur les opérations linéaires (deuxième note)."

(7) Professor F. MORLEY : "On an inversive relation between five points of a plane."

(8) Mr. J. E. WRIGHT : "Application of the theory of continuous groups to a certain differential equation."

(9) Dr. EDWARD KASNER : "Geometry of point correspondences : osculating homographies."

(10) Mr. C. H. SISAM : "On septic scrolls."

(11) Dr. E. V. HUNTINGTON : "Note on the definitions of groups, abelian groups, and fields."

(12) Dr. E. V. HUNTINGTON : "A set of postulates for ordinary complex algebra."

(13) Dr. BURKE SMITH : "On the deformation of surfaces of translation."

(14) Professor L. E. DICKSON : "A general theorem on algebraic numbers."

(15) Dr. A. B. COBLE : "The similar projective groups of a cubic space curve and a quadric surface."

(16) Professor E. H. MOORE : "On a definition of abstract groups."

M. Fréchet's paper was communicated to the Society by Professor E. H. Moore. In the absence of the authors, M. Fréchet's paper was read by Dr. E. B. Wilson, and the papers of Professor Dickson and Dr. Smith were read by title. Professor Dickson's first paper and the papers of Mr. Wright and Dr. Smith appeared in the January number of the BULLETIN. President Fiske's address is contained in the present number. Abstracts of the other papers follow below. The abstracts are numbered to correspond to the titles in the list above.

1. Dr. Mason proved that the necessary and sufficient condition that Poisson's equation in the plane

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f(x, y)$$

have a doubly periodic solution with the periods a, b, where f has these periods, is that the integral of f carried over the period rectangle of sides a, b is zero. A "doubly periodic Green's function" G was formed from sigma functions, its law of reciprocity developed, and the double periodic solutions of the differential equation expressed by quadrature in terms of f and G.

2. In this paper Professor Snyder completes the enumeration of sextic scrolls. Among the new forms added may be mentioned those having tacnodal curves of order 2, 3, 4, or 5, and a new form of developable, the latter having a cuspidal and a nodal quintic curve which have a common triple point. The method of point correspondence is used throughout; it is finally shown that if any two non-singular plane curves lie on an algebraic scroll, they must be projectively equivalent.

3. Dr. Coble's paper gives some applications of the following general theorem in the theory of forms: Any function homogeneous in the coefficients of each of a series of q-ary forms can be expressed *linearly* in terms of the coefficients of the concomitants of these forms. Hence interpreting a binary form of nth order as a point in a space of n dimensions, we may write down in terms of its concomitants of mth degree the most general spread of mth order. This permits of discussing easily the relations of such a spread to the *norm curve* in n dimensions. For example when $n = 3$ we derive the theorem: The most general algebraic surface of mth order which contains a cubic space curve as an asymptotic curve has $3(m - 2)$ double points on the curve.

An application is also made to the theory of syzygies. If the above surface of mth order written down in terms of the coëfficients of *all* the concomitants of mth degree contains more constants than the greatest possible number, the existence of one or more syzygies between the concomitants is clearly indicated. Thus we obtain an arithmetical criterion for the location of syzygies. The method may be extended to include the case of any number of q-ary forms.

6. Let U_f be a linear operation applied to the continuous function $f(x)$. It is first noted that the continuous functions $k_n(y)$ which occur in the formulas

$$U_f = \lim \int_a^b k_n(y) f(y) dy,$$

expressing Hadamard's theorem, can be chosen as polynomials. From this may be derived a new development of U_f. Another more interesting development is furnished by one of Borel's theorems. This development gives the value of U_f, nothing being known about f but its values for all rational values of x. It is proved that, U_f being a linear operation applied to a continuous function $f(x)$ defined between a and b, and c a number between a and b, two linear operations V_{f_1}, W_{f_2} [determined by the respective values f_1, f_2 of f in (a, c) and (c, b)] can be found so that $U_f = V_{f_1} + W_{f_2}$.

Some remarks are added on the importance of the field of functions in which a linear operation is defined.

7. If the reflexions of a point d in the sides of a triangle abc form a triangle perspective with abc, then the relation of a, b, c, d is a mutual one. Moreover the relation of $abcd$ and the point ∞ is a mutual one. The paper of Professor Morley is occupied with the form of this relation, with the curve (a bicircular quartic) on which the fifth point lies when four are given, and with the determination of the four points when the curve is given.

9. An equation of the form $y = f(x)$ may be interpreted on the one hand as a curve in a plane, and on the other hand as a correspondence between the points of a straight line. In the latter interpretation, the line is regarded as a two dimensional manifold, with the point pair as element. The geometry of correspondence is capable of as many developments as is the theory of curves; Dr. Kasner considers, in particular, the theory of contact. The first approximation to an arbitrary correspondence in the neighborhood of a point pair is given by a similarity transformation; the second approximation is given by an homography which may be said to osculate the given correspondence. When the variables are interpreted in the complex plane, this result is obtained: In connection with an arbitrary conformal transformation of the plane, there exists, for each point, a unique osculating circular transformation.

10. Mr. Sisam determined and classified the septic scrolls having a rectilinear directrix. A one-to-one correspondence is established by pole and polar theory between the generators of the scroll and the points of a twisted curve. The types of scrolls are then determined from the types of curves thus ob-

tained. The classification was obtained geometrically, but the method of obtaining the equations of the scrolls was also given.

11. Dr. Huntington's first paper gives a summary of all the known sets of independent postulates for abstract groups, and adds the following set of six postulates, which is in some respects simpler than any of the previous sets:

1. If a and b are elements of the class, then their "product," ab, is an element of the class.
2. The associative law holds: $(ab)c = a(bc)$.
3. There is at least one element i such that $ii = i$; and
4. There is not more than one such element. (This unique element i is called the "identity" of the group.)
5. If i is the identity (see 3 and 4), then either $ia = a$ for every element a or else $ai = a$ for every element a. (Either alternative can be deduced from the other, in view of 1–6, so that it is not necessary to demand both.)
6. If i is the identity (see 3 and 4), then for every element a there is either an element a'_r such that $aa'_r = i$, or else an element a'_l such that $a'_l a = i$. (Either alternative can be deduced from the other, in view of 1–6, so that it is not necessary to demand both. The elements a'_r and a'_l are shown to be uniquely determined by a, and to be equal to each other; this element $a' = a'_r = a'_l$, is called the "inverse" of a.)

These six postulates are sufficient for the theory of abstract groups in general; if we wish to make the groups abelian, we must add a seventh postulate, namely:

7. The commutative law: $ab = ba$.

All seven postulates are independent, and remain independent if we add the demand that the group shall be infinite. If, however, we demand that the group shall be finite, postulates 3 and 6 become redundant. These seven postulates for an abelian group lead at once to a set of fifteen postulates for a *field*; a condensed list of ten postulates is also given. The paper will appear in the *Transactions*.

12. Dr. Huntington's second paper shows that a class K, with a relation $<$ and two operations $+$ and $\times$, will form a system abstractly identical with the algebra of ordinary complex quantities, when the following conditions are satisfied:

I. The class K is a field with respect to $+$ and $\times$.

II. The class K contains a subclass C which is also a field with respect to $+$ and $\times$. (C corresponds to the class of reals.)

III. The subclass C is a one-dimensional continuum with respect to $<$ (see author's paper in the *Transactions* for January, 1905, § 1).

IV. Within the subclass C the following relations hold: 1) if $x < y$, then $a + x < a + y$; 2) if $a > 0$ and $b > 0$, then $a \times b > 0$ (where 0 is the zero element of the field).

V. There is an element j in K such that $j \times j = -1$ (where -1 is the negative of the unit element of the field); and if i is one of the elements which have this property, then for every element a in K there are elements x and y in C such that $x + iy = a$.

The postulates of these five groups, which number twenty-eight in all, are shown to be independent.

14. In his second paper, Professor Dickson considers the γ_{iks} of a multiplication table of a hypercomplex number system for the special case of algebraic numbers. Arranging the n^3 elements γ_{iks} in the form of a cube and taking the intersection by any plane parallel to a face, we obtain n^2 elements of non-vanishing determinant. This theorem that no one of the $3n$ determinants is zero is true for finite and algebraic fields, but not in general for other complex number systems. The problem arose in connection with an application to group theory.

15. By a method suggested by Professor Study, the integral equations of the projective three-parameter groups of a cubic space curve and of a set of generators of a quadric surface are obtained by Dr. Coble in such a form that the algebraic transformation T connecting them is evident. The effect of T upon the various manifolds is discussed and the results are used to obtain some of the properties of the algebraic group reciprocal to the projective group of the cubic curve.

16. Professor Moore improves the second of his two definitions of abstract groups given in the *Transactions* of October, 1902, by proving that the postulate $(3'_r)$ is redundant and that the remaining postulates $(1, 2, 3,'' 3''_l, 4''_l)$ are mutually independent.

F. N. Cole,
Secretary.