

After considering the graphs of some elementary functions, the idea of limit and limiting value of a function is introduced and a continuous function defined by the relation, $\lim_{x \rightarrow a} f(x) = f(a)$. In the theorems on limits, in preparation for derivative, the limit of $\frac{\sin \theta}{\theta}$ and of the series for e are taken up.

No rigorous consideration of the limit of $\left[1 + \frac{1}{z}\right]^z$ is given, but the quantity is calculated for ten values of z and the graph constructed as illustration of the fact that the limit is the previously defined quantity e . Exercises follow on the limiting values of some elementary functions.

Chapter II. gives a clear conception of derivative, the general definition being given after the consideration by text and examples of increment and increment quotient, and the special cases of velocity and slope of the tangent of a curve. General rules, the derivatives of the elementary functions and numerous examples follow.

Chapter III. takes up tangent and normal, maxima and minima, expansion of functions and differentials. Maxima and minima are treated very clearly. The theorem of the mean is stated, the proof being geometric. As a more general law of the same nature Taylor's expansion with remainder is given, but without proof. Differential of $f(x)$ is defined as the first term in the expansion of $f(x + \Delta x) - f(x)$. In the exercises the meaning of differential and infinitesimal of higher order is brought out by examples of area and volume.

Chapter IV. takes up the definition of integral, integrals of the elementary forms, definite integrals and limit of a sum. Chapter V. is devoted to functions of two or more variables, partial derivative, total differential and total derivative.

The press work on the book is good and the page — very different from the average English text — looks interesting. The book should meet with success, for by its aid the "short course" may become really worth while. MAX MASON.

Geometrische Transformationen. I Teil: Die projektiven Transformationen nebst ihren Anwendungen. Von DR. KARL DOEHLEMANN. Leipzig, Göschen, 1902. vii + 322 pp.

THE theory of transformations has come to be of fundamental importance in geometry and yet the number of books devoted

to a systematic treatment of the theory is very limited. Under the title of geometric transformations, Doehlemann proposes to develop the properties of those transformations which are susceptible of a purely geometric treatment. This line of demarcation is, as the author recognizes, not very definite; but it excludes, for example, the consideration of the totality of conformal transformations in the plane. The volume at hand is devoted to projective transformations and will be followed by a second volume dealing with Cremona transformations and higher correspondences.

The volume is divided into four parts dealing with projective coördinates (pages 1–64) and the projective transformations on the line (pages 65–105) in the plane (pages 106–247) and in space (pages 248–320). The geometric point of view is kept in the foreground, as the avowed object of the book is to increase the pleasure of geometric studies, although analytic methods are used freely. Many applications are considered and, in particular, considerable attention is devoted to the applications to descriptive geometry.

The chapters on projective coördinates require but little comment. Various properties of cross ratios are derived and then the projective coördinates of points of a range and lines of a pencil are defined by the use of the cross ratio. The parametric representation is considered and the angle between two lines expressed as a cross ratio. The treatment of coördinates of points and lines in a plane is excellent and the discussion of the coördinates in space is as detailed as that for the plane.

A projective transformation of a given fundamental configuration into a second of the same number of dimensions, as the points of a plane into the lines of a second plane, is defined by putting those elements in correspondence whose coördinates are proportional. Each of the last three parts of the book opens with a discussion of the transformation thus defined, which leads to the invariance of the cross ratio and the number of pairs of corresponding elements necessary to determine the transformation. Then comes the discussion of the general linear transformation and some attention is paid to the degenerate cases in which the determinant of the transformation is zero. In each of these three parts the transformation of one fundamental configuration into a second is completely discussed before the transformation of a configuration into itself is considered.

The parametric representation of a one-dimensional configuration is developed at length and much attention is paid to cyclic projectivities and, in particular, to involution. In the plane and in space the Möbius net is used considerably and correlations as well as collineations receive their due share of attention.

In dealing with the transformation of a line, a plane or of space into itself the nature of the invariant configuration is not developed with as much completeness as it might have been, nor are the canonical forms of the equations of the transformations as thus classified given even for the case of the line.

The group idea is not considered at all although the word group is used twice, parenthetically. It would seem reasonable that, in a book of over three hundred pages devoted to the consideration of projective transformations only, space might be found to show at least the relations which the euclidean and projective geometries bear to their respective groups.

The appearance of the book is sufficiently described by saying that it appears in the *Sammlung Schubert*. There are some typographical errors, of which those noticed as most apt to confuse the student may be mentioned here. On page 8 in the definitions of h_1 and k_2 the e_1 and e_2 should be interchanged; on page 71, line 14, for 5 read 7; on page 76, line 1, for 40 read 11; and on page 87, line 15, b'_1 and b'_2 should be interchanged. An error appears on page 92: the third line from the bottom should read

$$c = 0, a + b = 0 \text{ and } b + d = 0,$$

so that the last equation on the page should be

$$\lambda\lambda' - \lambda + 1 = 0.$$

It is then unnecessary to make the change in parameters on page 93 which shows the identity of the two cyclic projectivities under discussion.

The book contains much that is interesting and helps to fill a decided gap in the works on geometry.

ARTHUR SULLIVAN GALE.

Aufgaben aus der niederen Geometrie. Von IWAN ALEXANDROFF, mit einem Vorwort von Dr. M. SCHUSTER. Leipzig, Teubner, 1903. vi + 123 pp.

ANYTHING tending to systematize the solution of the "originals" of elementary geometry is welcomed as an aid to