such a way as to preserve a constant ratio between corresponding areas. The theory of the large Prussian maps (Messtischblätter) with linear scale of 1 to 25,000 units is discussed, and elaborate calculations are given to show the variation between the lengths of north and of south boundaries between two common meridians.

The second chapter begins with a much more theoretical discussion of two spherical representations, that in which areas are preserved, and the conformal. The maximum difference between the spherical and the geographic latitude is determined. About forty pages are then devoted to the double conformal representation, first of the spheroid upon the gaussian sphere, then the latter upon the plane by Mercator's projection, the curve of contact of the sphere and cylinder being the principal meridian. The process is illustrated by actually determining four points upon the plane when their spheroidal coördinates are known by triangulation and astronomical observation. The calculation covers ten pages, every detail being given and all the numerical work being done with 7-place logarithms.

The book is almost free from typographical errors. On page 50 is a slip; an elementary integral is there called elliptic. The sections are marked by bold-faced type, but only named in the table of contents. The book is provided with an index.

VIRGIL SNYDER.

Die Horopterkurve, mit einer Einleitung in die Theorie der kubischen Raumkurve. Von Dr. WALTHER LUDWIG. Halle, Schilling, 1902. 36 pp.

THIS monograph was prepared to explain the new models of twisted cubic curves (Schilling's catalogue, series XXVII, number 6). Until recently the only models of these curves in the Schilling collection were those on plaster cylinders, which on account of the opaqueness of the material and the smallness of the scale conveyed but little information.

The present series consists of six pieces; four represent the standard forms of the curve traced on celluloid cylinders about twenty inches high, the fifth is the developable of class three made of silk threads and the sixth is a heavy wire model of the horopter.

The memoir is divided into two parts, the general discussion of the cubic curve, and the particular discussion of the horopter. The first part is very elementary. It begins with a discussion of projective pencils of planes. Then the cubic curve is shown to be the locus of intersection of three corresponding planes in three projective pencils. The generation of the same curve as locus of intersection of corrresponding lines of two projective bundles is treated more briefly and less clearly, although it is the only generation with which we are concerned in the second part. The usual classification of the curves according to the configuration on the infinite plane completes the first part.

The second part begins with the physiological principles underlying the horopter and their geometric meaning. The locus of the point which appears as a single point for a fixed position of the eyes is called the horopter of this position. In order to obtain the form of this curve various theorems are assumed which are only established empirically — those relating to the center of vision and center of rotation. The images on the retinas, connected with the center, form two projective bundles of rays. When two corresponding rays intersect, the point which they project is seen simply. The case considered is that in which the head is vertical, the line of sight horizontal and directly in front of the person.

The desired horopter is therefore a twisted cubic. It can be defined in terms of a parameter t as follows:

$$x = \frac{-2b \sin \psi t}{t^2 + 2t \cos \psi + 1}, \quad y = \frac{b(1 - t^2)}{t^2 + 2t \cos \psi + 1}, \quad z = \frac{c(1 - t)}{1 + t}.$$

It lies on the cylinder of revolution

$$x^2 + y^2 - 2bx \cot \psi - b^2 = 0.$$

Since two points on each secant of the curve are seen simply, it follows that every chord of the horopter appears as a simple line, though its various points may appear double. The curve passes through K_1, K_2, the vertices of the two projective bundles. The finite arc between K_1, K_2 has no physiological meaning.

In the model the curve itself is made of brass — a line of symmetry and an asymptote are of nickel. Two little spheres represent K_1, K_2; from these pass two copper wires to a third sphere on the curve. The copper wires represent lines of sight. Two white lines on the base of the stand supporting the curve indicate the intersections with this plane of the frontal and median planes.

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