SHORTER NOTICES.

Étude de quelques surfaces algébriques engendrées par des courbes du second et du troisieme ordre. Par M. STUYVAERT, Professeur à l'Athenée Royale de Gand. (Thèse). Gand, Hoste, 1902. 72 pp.

CONSIDERING surfaces generated by a curve, plane or twisted, as a logical generalization of the ruled surface, Stuyvaert discusses surfaces generated by a conic or a twisted cubic. His style is clear and concise and he arrives at many known results in a new way and gives many new and interesting ones. The dissertation is divided into three chapters. Chapter I discusses surfaces which are generated by a plane which contains a conic having six points on a directrix curve or system of directrix curves. The first case considered is that of a single directrix C_m, a unicursal curve of order m. The class of this surface is determined as follows. Let S_2 be a quadric passing through four fixed points, A, B, C, D, and cutting C_m in $2m$ points, and let π be a plane through a general line d, cutting C_m in m points. The class of the surface will be the number of groups of six points common to the planes through d and the quadrics through A, B, C, D. The quadrics mark on C_m an involution I_5^{2m}, the planes mark an involution I_1^m. The number of groups of six points common to the two is expressed by

$$C_1^{2m-5} \cdot C_5^{m-1} = (2m-5)C_5^{m-1}.$$

But in each of the four planes (dA), (dB), (dC), (dD), there are C_5^m groups of six points situated on degraded quadrics and hence should be excluded, therefore the class is given by

$$\nu = (2m-5)C_5^{m-1} - 4C_5^m.$$

By essentially the same method the class of surfaces whose directrix curves consist of a line and a C_m, a conic and C_m, etc., is obtained. As corollaries of the preceding theorems follow readily the determinations of the orders of surfaces generated by ∞^1 conics which move on a system of directrices. In this connection the theorem is given: *Conics whose plane passes through an axis $\alpha\beta$, and which cut five independent lines generate a surface of order eight.*

In chapter II surfaces generated by ∞^1 conics are discussed. First it is shown that the order of a surface generated by conics whose planes pass through an axis $\alpha\beta$ and which cut each of five unicursal directrices in one point depends upon the order of the surface which has five rectilinear directrices. This last surface is shown to be of order eight and is denoted by S_8.

The analytic discussion of the surface S_8 is then taken up. It is unicursal; for if through a point M of the surface a line be drawn cutting the axis $\alpha\beta$ and one directrix, it will pierce an arbitrary plane π in a point P, the image of M. Conversely a line drawn through P cutting the axis and one directrix cuts the surface in a single point M. The surface can therefore be depicted on the plane rationally. Two parameters can therefore be chosen, in terms of which the surface can be expressed rationally. The first is ω, so chosen that

$$\alpha_x + \omega\beta_x = 0$$

represents the plane $(\alpha\beta M)$, α_x, β_x being planes through the axis $\alpha\beta$. The second parameter is the anharmonic ratio λ of M, C, C', C'' where the generator through M cuts three directrices. In terms of these two parameters the coördinates of the surface are rationally expressed in terms of functions of degree seven in ω and degree two in λ. The remainder of the chapter is concerned principally with the properties of this surface and its plane depiction. Among others this interesting theorem is derived: *Every quadric passing through a generating conic of S_8 is tangent to the surface in four points of this conic.* This is an extension of a corresponding theorem for ruled surfaces.

Chapter III deals with sheaves of twisted cubics. A sheaf which contains the sheaf of Reye and the sheaf of Sturm as special cases are discussed in considerable detail. The sheaf is defined by

$$\alpha a_x/b_x = \alpha' a'_x/b'_x = \alpha'' a''_x/b''_x$$

where a_x, b_x, $a'_x \cdots$ are planes and α, α', α'' are arbitrary constants. It is seen that the cubics have in common three bisecants and two points. These equations will represent the sheaf of Reye, consisting of all the cubics passing through five points, if

$$|aba'b'| = |aba''b''| = |a'b'a''b''| = 0.$$

They will represent the sheaf of Sturm, consisting of all the cubics having the same osculating tetrahedron, if

$$a'_x \equiv b_x \quad a''_x \equiv b'_x.$$

Surfaces generated by ∞^1 of these cubics, *e. g.*, all those cutting a fixed line, touching a plane, etc., are discussed at some length. The plane depiction of these surfaces is also taken up. This chapter takes up nearly half the paper. Many interesting theorems concerning this sheaf are proved, also the corresponding ones for the sheaf of Reye and of Sturm are given. Several problems are suggested in the course of the discussion, some of which the author states he expects to solve.

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Das Erdsphäroid und seine Abbildung. Von Dr. EMIL HAENTZSCHEL, Professor an der kgl. technischen Hochschule und am köllnischen Gymnasium zu Berlin. Leipzig, Teubner. 1903. viii + 140 pp., 16 figures.

THE purpose of this book is to discuss the practical problems of map drawing. It differs from many other works on the subject by leaving aside all those problems which are only of theoretical interest, and by including most of the numerical calculation of those considered. The author makes no claim for completeness, but still he presents enough of the subject to make his problem of the actual construction of geographical maps entirely intelligible. The book is very full of references to more extensive treatments of each particular problem discussed. A knowledge of the relations between exponential and trigonometric functions and of the elements of analytic geometry and the calculus is presupposed, although most of the formulas are derived with great detail. An introduction presents the evidence for the spheroidal form of the earth; it is assumed to be of revolution and Bessel's constants are used. The author mentions that probably Clarke's determination is more accurate than Bessel's.

The first chapter discusses the relations between the various kinds of latitude, geographic, geocentric and reduced (eccentric angle), and the determination of the maximum difference between them. The length of a degree along a meridian is fully discussed and it is clearly shown why a knowledge of its length is valuable. The area of a zone defined by two parallels of latitude is shown not to depict on the concentric sphere in