The congruence is of order 3 and class 2, dual to the congruence of axes in a linear sheaf of complexes. The discussion of the distribution of the lines on ∞^1 coaxial hyperboloids and their association to the conjugate congruences (*i. e.*, such that every complex of the first sheaf is in involution with all those of the second) is followed by a more thorough study of the parametric hyperboloids themselves. The pedal surface is coincident with the mean surface; it is a particular form of the Steiner surface. The focal surface is the same for both congruences; it is of order 6 and class 4. This surface is exhaustively discussed, first geometrically, then by means of elliptic functions.

After the treatment of the general case is completed, each particular case is fully discussed. In these 70 pages is found not only a systematic and far-reaching discussion of the configuration named, but also a large number of other problems suggested that might furnish themes for further investigation. The treatment differs essentially from that of other authors in the emphasis put upon particular and degenerate forms. While others confine their attention to the so-called "general case" — a case which in many applications does not exist at all — the present work concerns itself much more with each particular type, so that in the applications no exceptions are presented.

In conclusion we would say that the author has produced a work that will for some time furnish material for study and reflection. The style is not inviting, and the treatment is frequently involved, but the author has presented a new field of mathematics. Many commentaries will be necessary to make the work generally accessible, but they will surely come.

VIRGIL SNYDER.

CORNELL UNIVERSITY,
October 12, 1903.

ENCYKLOPÄDIE DER ELEMENTAR-MATHEMATIK.

Ein Handbuch für Lehrer und Studierende. Von HEINRICH WEBER und JOSEF WELLSTEIN. Erster Band. Elementare Algebra und Analysis. Von H. WEBER. Leipzig, Teubner, 1903. xiv + 447 pp.

IT was with much interest that scholars learned, a few years ago, of the proposed appearance of an Encyklopädie der Mathematischen Wissenschaften, and this under circumstances to assure the highest standard of excellence. The publication of this monumental work was begun in 1898, and the parts which have thus far appeared have in general fulfilled the expectations of mathematicians. It has been international in its view and in its contributions, rich in its bibliography, and comprehensive in scope. It has, however, failed to touch the field of elementary mathematics in such a way as to be helpful to the great body of secondary teachers and students.

Recognizing the need for a more elementary work, the same publishers have undertaken the Encyklopädie der Elementar-Mathematik, originally planned for two volumes, but now announced to include a third on the applications of the subject.

The general scheme is quite as commendable as that of the more pretentious work in the higher fields, for students and teachers in elementary lines are even more in need of helpful suggestions as to bibliography, modern methods, and the improvement of their subjects from the standpoint of recent mathematics, than are the investigators in the advanced theory. A difficulty presents itself, however, in defining the term "Elementary mathematics," and here the authors confessedly lay themselves open to criticism. They admit that all attempts at fixing the limits on purely mathematical grounds have been unsatisfactory, and they have been compelled to resort to educational considerations instead. It must be confessed that the result has not been altogether satisfactory, and it is difficult to reconcile it with the announced position taken by the authors. Why analysis should include infinite series and products and the transcendental nature of e and π, but contain no reference to even the elements of differentiation and integration, is not apparent, and why so many of the common topics of the elementary mathematics found in every secondary school should be omitted altogether, is equally inexplicable.

The present volume is divided into three "books," the first treating of arithmetic, the second of algebra, and the third of analysis. The term arithmetic is used in the German sense, and thus includes what in other countries is quite generally designated as elementary algebra. Nearly two hundred pages are devoted to this subject, divided among the following topics: Natural numbers, the fundamental operations, fractions, irrational numbers, ratio, powers and logarithms, equations of the first and second degrees, imaginary numbers, permutations and combinations, and various applications to series.

The second book, on algebra, includes the following topics: Algebraic equations, the leading propositions of algebra, indeterminate linear and quadratic equations, continued fractions, the cubic and quartic equations, approximations of roots of numerical equations, cyclotomic equations, and a discussion of certain cases impossible of solution, as of the cubic by square roots alone, and of the quintic by radicals.

The book on analysis considers infinite series and products, including the question of convergency, and gives proofs of the transcendence of e and π.

The most interesting portions of the book on arithmetic are those devoted to the origin of the number concept, the theory of the negative, the Dedekind theory of the irrational, the Du Bois-Reymond theory of ratio, and the combinatory theory, including a few of the fundamental propositions of groups.

Several sections of the book on algebra are of especial interest. The existence theorem for roots, for example, is treated by the first (1799) of Gauss's methods (as improved in 1849), the presentation being exceptionally clear. Indeterminate linear equations form the basis for a brief treatment of number congruences, and indeterminate quadratics serve in a similar way for an introduction to the study of primes. The Galois theory of equations is presented in connection with the equation of the fourth degree, although no reference is made to its application to the quintic equation. The cyclotomic equation is especially well discussed, including Gauss's treatment of the regular 17-gon. The chapter on the impossibilities of algebra is also one of the most valuable in the whole work. One of the strange bits of conservatism is the author's failure to mention Horner's method of approximation, although, unlike many of the older continental writers, he is by no means ignorant of it.* Instead of making use of it in the Encyklopädie, he follows the usual German method, through Sturmian functions, the Regula falsi, and the Newtonian approximation.

As would naturally be expected, the work shows on every page the German accuracy of proof, and the name of Teubner is an assurance of a good piece of bookmaking. On these two points there can be no reasonable criticism. Indeed, the general appearance of the page is distinctly better than that in the

* Vide his Algebra, vol. I, p. 382.

larger Encyklopädie, even in respect to the mediocre figures. The latter are, however, below the general standard of the book, a capital in the figure sometimes appearing as a small letter in the text, a small x standing for both a point and a line, and the origin being indifferently O and o. Of misprints there seem to be few, although "solchcn" (page 91) and "hypotonuse" (page 315) show that they exist.

From the outline already given it may rightly be inferred that the work is arranged and written rather in the style of a higher algebra than an encyclopædia in the English or French sense. It is certainly in no respect a *Lexikon*, even with its excellent index, and on the whole, by those who looked for a book of general information on elementary mathematics, it will be considered disappointing. But aside from the technical use of the name, which is quite correct from the Teutonic standpoint, one still has good cause for disappointment. He looks for a general and exhaustive treatise, and finds only a somewhat condensed higher text-book; he looks for a wide range of topics, and finds only a partial list of even the essentials; he looks for help in the way of bibliography, but he finds only a few commonplaces; he has hopes of an elementary work made up like Pascal's Repertorium, but he finds one more closely resembling Pund's Algebra. A teacher of the elements might reasonably expect, for example, to find some systematic treatment of factors in an encyclopædia of mathematics; he might hope for some suggestions on the subjects of graphs or undetermined coefficients; and he might feel that here he could have light thrown upon some of the vexing questions of extraneous roots, radical equations, and the exact meaning to be attached to certain symbols. He might also hope for some adequate treatment of elementary determinants, possibly a few suggestions as to the theory of numbers, and very likely a brief treatment of a topic like numerical computations. None of these topics will he find discussed, however, and some are not even mentioned.

It might also be expected that some efforts would be made to supply historical notes of interest to one who is teaching or reviewing the subject, even if not of value to one engaged in research. Such notes as appear are, however, practically worthless. Under complex numbers, for example, the names of the greatest contributors, save only Gauss, are wanting, while the only name mentioned in connection with the Delian problem is Plato, and the assertion that Joost Bürgi first used a decimal point is altogether too emphatic. In connection with the cubic equation, Cardan's formula is referred to his Practica arithmeticæ generalis (1537) instead of the Ars magna (1545), an error due to misreading Cantor. Furthermore, not only are the historical references very meager, but they are confined almost exclusively to the German, and even then too exclusively to Cantor. For example, a work that pretends to be at all international might be expected to mention Heath's Diophantos in connection with a note upon the editions of this writer's works, though why these editions should be given at all in a work so barren of more important references, is a question.

A treatise of this kind might also be expected to furnish a good working bibliography, in no sense exhaustive, but suggestive and helpful. The bibliographical notes which Professor Weber has inserted are, however, with slight exception, of no practical value, and are evidently selected with no well defined purpose.

With all due appreciation of the scholarship of the work, and of its helpfulness, it must therefore be a matter of regret to all who have looked forward to its appearance, that the ground covered is not that of elementary mathematics in an international sense, that the historical notes are very ill considered, that no attempt has been made to offer a helpful bibliography, and that the arrangement and general treatment are so far removed from that of the Repertorium or the Burkhardt-Meyer Encyklopädie. DAVID EUGENE SMITH.

SHORTER NOTICES.

Mathematical Papers of the late George Green. Edited by N. M. FERRERS and published by Macmillan in 1871. Facsimile reprint. Paris, A. Hermann, 1903.

GREEN'S celebrated paper on potential, published at Nottingham in 1828 by private subscription, remained practically unknown for many years. At this period George Green was entirely self-taught and had no more advantages than a provincial town with but few mathematical works of any kind was likely to furnish. He was, however, "discovered" and sent to Cambridge in 1833, taking his degree in 1837 and his fellow-