of the fourth harmonic to two points of this envelope and the point of intersection of the secant through these two points with the plane of the singularities of the surface.

YALE UNIVERSITY,
October, 1903.

ON THE GENERATION OF FINITE FROM INFINITESIMAL TRANSFORMATIONS — A CORRECTION.

BY PROFESSOR H. B. NEWSON.

(Read before the Chicago Section of the American Mathematical Society, January 2, 1903.)

IN a paper entitled "Continuous groups of circular transformations" which the author read before this Society, April 24, 1897, and which was published in the BULLETIN (2) series, volume 4, pages 107–121, there occurs a serious error * which I desire to correct.

The error in question is a misstatement of the number of logarithmic spirals of the family $\rho = e^{(c+i)\theta}$ (where c is a parameter) that pass through a given point of the plane. It was stated on page 114 of the above mentioned paper that in general only two spirals of the family pass through a given point. In fact there are an infinite number of these spirals through a point P.

To show this let, the coördinates of P be $(\rho_1, \theta_1 + 2n\pi)$. Since $\rho_1 = e^{(c+i)(\theta_1 + 2n\pi)}$, then

$$\log \rho_1 \equiv \log r + 2im\pi = c(\theta_1 + 2n\pi) + i(\theta_1 + 2n\pi);$$

whence $\log r = c(\theta_1 + 2n\pi)$ or $c = \log r/(\theta_1 + 2n\pi)$. Since n is any integer, c may have any one of an infinite number of values. Thus there are an infinite number of spirals of the family through the point $(\rho_1, \theta_1 + 2n\pi)$. When $n = 0, 1, 2, 3, \cdots$ the corresponding spiral, starting from the origin, makes $0, 1, 2, 3, \cdots$ turns about the origin before passing through the point P.

The last paragraph on page 114 and the first on page 115, including theorems 7 and 8, of the above-mentioned article should be corrected to read as follows:

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* My attention was first called to this error by Professor Frank Morley.

Different values of c give us different spirals, each of which corresponds to a one parameter subgroup of G_{mn}. c varies continuously through all real values from $-\infty$ to $+\infty$, so that these spirals lie infinitely close to one another. They all pass through the unit point. As c approaches zero, the corresponding spiral approaches as a limit the circle of unit radius about the origin; as c approaches infinity the corresponding spiral approaches as a limit the straight line which is the axis of real numbers.

Every point in the plane not on the unit circle lies on an infinite number of discrete spirals, from which we infer that every loxodromic transformation in G_{mn} belongs to an infinite number of distinct one-parameter subgroups. Every hyperbolic transformation in G_{mn}, except the involutoric transformation, belongs to an infinite number of loxodromic subgroups as well as to the hyperbolic subgroup. The elliptic transformations in G_{mn} belong only to the elliptic subgroup. The involutoric transformation is common to the elliptic and the hyperbolic subgroups; the identical transformation is common to all the subgroups; the two pseudo-transformations are common to all except the elliptic subgroup. Any two loxodromic subgroups have an infinite number of discrete transformations in common.

THEOREM 7. *Every one-parameter subgroup in G_{mn} is continuous. Every non-elliptic transformation in G_{mn} belongs to an infinite number of distinct subgroups.*

This same geometric representation enables us to discuss intuitively the generation of finite transformations by the repetition of infinitesimal transformations. Every spiral passes through the unit point, and corresponding to the two points on the spiral adjacent to the unit point we have two infinitesimal transformations belonging to a one-parameter group. These are given by $k = e^{+(c+i)\delta\theta}$ and $k = e^{-(c+i)\delta\theta}$. The identical transformation divides the one parameter group into two portions, each of which contains an infinitesimal transformation. Every finite transformation in each portion of a one parameter loxodromic group can be generated by the repetition of the corresponding infinitesimal transformation. In the elliptic group for which the spiral reduces to a circle, every transformation can be generated from either elliptic infinitesimal transformation. In the hyperbolic group, for which the spiral reduces to the axis of real numbers, the transformations for which k is negative can not be generated by the repetition of either of the hyperbolic infinitesimal transformations. Every non-elliptic transformation in G_{mn} can be generated from an infinite number of distinct infinitesimal transformations; for every such transformation belongs to an infinite number of distinct subgroups.

THEOREM 8. *Every finite transformation of the group* G_{mn} *can be generated by the repetition of an infinitesimal transformation of the group. Every elliptic (non-elliptic) transformation in* G_{mn} *can be generated from two (an infinite number of) distinct transformations. Hyperbolic transformations for which* k *is negative can not be generated by either hyperbolic infinitesimal transformation.*

UNIVERSITY OF KANSAS,
February, 1903.

STUDY'S GEOMETRY OF DYNAMES.*

Geometrie der Dynamen. Die Zusammensetzung von Kräften und verwandte Gegenstände der Geometrie. Von E. STUDY. Leipzig, Teubner, 1903. 8mo., 603 pp., 46 figs.

THE original purpose of Professor Study's book was to present a systematic geometric treatment of the composition of forces acting on a rigid body, but as the work progressed the idea broadened, resulting in an elaborate treatise on a whole field of geometry, hitherto nearly unknown.

Probably no other work on geometry that has appeared since the memoirs by Klein and by Lie in the early volumes of the *Mathematische Annalen* contains so many original and fruitful ideas as that under review.

The book is divided into three parts: the first (pages 1–122) treats of the composition of forces as a problem in pure geometry; the second (pages 123–225) treats the same problem analytically, making free use of symbolic notation and of line geometry; the third (pages 226–556) is devoted mainly to the discussion of a transformation first met with in the second part.

The first two parts contain a new geometric theory of the composition of forces and of infinitesimal motions. In the first part no use is made of other branches of mathematics than elementary geometry and trigonometry, but in order to express

* Dyname is the form used by Plücker in his English papers in the London Transactions Roy. Soc.