may almost say necessary. There may be other books as good; but for this particular purpose these are not easily improved upon. EDWIN BIDWELL WILSON.

ÉCOLE NORMALE SUPÉRIEURE,
PARIS, FRANCE,
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SHORTER NOTICES.

Urkunden zur Geschichte der Mathematik im Mittelalter und der Renaissance. By M. CURTZE. Zweiter Theil. *Abhandlungen zur Geschichte der mathematischen Wissenschaften,* XIII Heft. Leipzig, Teubner, 1902. 292 pp. 14 marks.

Abhandlungen zur Geschichte der mathematischen Wissenschaften, XIV Heft, Leipzig, Teubner, 1902. 338 pp. 16 marks.

THE first part of Curtze's Urkunden, forming the twelfth volume of the Abhandlungen, has been reviewed in the BULLETIN * so recently that it is quite surprising to find two new volumes of the series already published. Indeed no better evidence of the present revival of interest in the history of mathematics can be found than is seen in the encouragement recently given to this series founded a quarter of a century ago by Professor M. Cantor. The publication of the first seven volumes extended through a period of nineteen years, while the last seven, including the two under review, have appeared since 1897.

The second part of Herr Curtze's Urkunden is devoted to two interesting manuscripts, one the Practica Geometriæ of Leonardo Mainardi of Cremona, and the other the algebra of Initius Algebras. The first, which also bears the title Leonardi Cremonensis Artis Metrice Practice Compilatio, is a transcript, with German translation, from an Italian codex in the Venetian dialect in the university library at Göttingen. This codex is not unique, for Prince Boncompagni had two Latin manuscripts of the same work; but not only has it never before been published, but Leonardo Mainardi has been practically unknown to historians of mathematics. It consists of fifty folios, of which the first twenty-nine and the last fourteen are here

* Vol. 9, p. 123, Nov., 1902.

transcribed and translated, the others containing merely tables of sines and a *tabula sollis* (sic), that is a table of the lengths of the various days of the year. Of the Boncompagni codices, the older belongs to the fifteenth century, although ascribed by Narducci to the fourteenth — a century before Leonardo was born. The Göttingen codex, which Herr Curtze has carefully compared with the other two, was written in 1488.

Of the life of Leonardo nothing is known save what appears in the Cremona Literata of Franciscus Arisius, published in 1702, who seems to have had access to the later of the Boncompagni codices, and who speaks of "Leonardus Maynardus Insignis Astronomus, Physicus et Mathematicus," as having flourished in LXXXVIII, that is in 1488.

The treatise itself is not on pure geometry, but is devoted to the mensuration of heights, surfaces and solids, with such trigonometry as was necessary for the work in hand. It is divided into three "tractati," the first being introduced with an apology for entering a field already well covered, but with the hope that this "picholo volumen" may prove a valuable compendium. This tractate is devoted to the measurement of heights, depths and lengths; the second discusses polygons and the circle, with their superficial measurement; and the third relates to the volumes of polyhedra and the three round bodies. Of the quaint mathematical terms, which of course include *umbra recta* and *umbra versa*, one of the most interesting is *columna uniforme* for a right circular cylinder or prism, a name with much to commend it. The common rules are, of course, substantially our present formulas, except for such interesting variations now and then as that for the volume of the sphere, $\frac{1}{4}d^2\pi\ \frac{2}{3}d$. In general the value of π is taken as $3\frac{1}{7}$, but in one case he gives the interesting approximation $3\frac{39831}{282296}$. This he obtains from the Archimedes limits by this process: $3\frac{10}{71} + \frac{1}{8}(3\frac{1}{7} - 3\frac{10}{71})$, and adds, "e questa e la proporcion del diametro a la circonferentia," interesting as giving the older use of "proportion," as a close approximation, as indicating that Leonardo thought this the exact value, and as inverting the terms of the ratio.

The second part of Herr Curtze's work is entitled "Die Algebra des Initius Algebras ad Ylem geometram magistrum suum." Of this work, almost unknown * to historians, four

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* See Cantor, *Vorlesungen über Geschichte der Math.*, vol. 2, p. 589.

manuscripts are extant, no one of them the original. Of these the one in the university library at Göttingen, a manuscript of two hundred and five folios, has been followed by Herr Curtze, but it has been carefully compared with the others (all in the Dresden library) and any material differences have been noted. The book is exceedingly interesting on several accounts, mathematically and historically. Written in the south German dialect in the sixteenth century, it displays considerable knowledge together with a confusion of historic facts, and shows no little acquaintance with the algebra that was just beginning to be revived in Italy. The prologue opens: "Hic hebet sich an das Buch Algebræ,* des grossen Arismetristens, geschrieben zu den zeithen Alexandri vnd Nectanebi, des grossen Grecken vnnd Nigromantis, geschrieben zu Ylem, dem grossen Geometer jn Egypten, jn Arabischer Sprach genant *Gebra vnnd Almuchabola*, das dann bey vns wirdt genant *das Buch von dem Dinge der vnwissenden zall.* Vnd ist aus Arabischer Sprach jn kriechisch transferirt von Archimede, vnnd aus kriechisch jn das Latein von Apuleio, vnd wird genandt bey den Welschen *das Buch de la cosa*, das dann aber wird gesprochen *das Buch von dem ding* * * * Vnnd aus disem Buch finden wir, das der Machomet in seinem Alkoran vermeldet von disen Regeln, vnnd nennet sie auch Gebram vnd Almuchabolam. Sie werden auch gebraucht von den Indiern * * * Sie gaben auch das gemelte Buch durch etliche grunde geleutert, als es Algebras gesagt hatt, vnnd ist geschrieben erstlichen von Algebras zu Ylem, dem grossen Geometer, der do was preceptor oder vorfarn Euclidis des fursten zu Megarien."

This passage is one of the most interesting, for its length, in the history of mathematics. The unknown Initius Algebras appears as one who lived in the fourth century B. C., contemporary with the equally unknown Ylem. The title of Alkhowarizmi's book is given, but the Arabic word *algebra*, which there first appears, is assigned to a period over a thousand years earlier and to the Greek language. Mohammed the prophet is confused with the son of Moses, ben Musa Alkhowarizmi, and the not uncommon mistake is made of taking Euclid of Megara for Euclid the Alexandrian. The early German use of "Welsch," which appears in the "Welsch practice" of the sixteenth century arithmetics, is seen; the

* *I. e.*, the author, Initius Algebras.

uncertainty of name for the comparatively new science is apparent; the "unknown quantity" takes its place in algebra; and India is recognized as a mathematical center. Interesting as is the passage, it is no more so than many others in the work, although no other condenses so much in an equal space.

The work covers the ground of the Arab algebra, with the usual German symbolism of the sixteenth century. But the most noteworthy feature of the entire text is the reference to the solution of the cubic. That the author thought he had a solution is apparent from two of his statements,* in the second of which he expressly says that he will later show how to solve equations of the (modern) forms,

$$a + bx^2 + cx^3 = dx, \quad cx^3 + a = dx + bx^2, \quad cx^3 + dx = bx^2 + a.$$

To be sure this promise is not fulfilled, but for such a statement to be definitely made, by a writer who knew a great deal of algebra, and made before the Ars magna of Cardan appeared, is at least very interesting. It suggests a possibility that, after all, the Tartaglia-Cardan formula was only a rediscovery of some cherished secret of a forgotten Arab writer whose work appears in unrecognizable form in this German manuscript. Such an interpretation, at least, seems to be in Herr Curtze's mind, although to most readers it will doubtless seem more probable that the wish of the writer was father to the thought, and that it was only a dream like that of the alchemist or the circle squarer, that found place in the mind of the algebraist.

The fourteenth volume of the *Abhandlungen* contains three articles. Of these the first is by A. A. Björnbo, entitled "Studien über Melenaos' Sphärik; Beiträge zur Geschichte der Sphärik und Trigonometrie der Griechen," and served in part as the author's inaugural dissertation in 1901. It forms without question the most complete study of Menelaus that has yet been made, not only from the sources already published, but from five of the best-known Latin manuscripts of the Sphere, no Greek copy being extant, and from Arab sources as well. The author has also collated the references to Menelaus by the various Greek and Arab writers, and has critically compared the various printed editions of the work in hand, all with a scholarship that promises well for his future work in the history of mathematics.

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* Pages 400, 540.

The well-known relation of the Sphere of Menelaus to the plane geometry of Euclid is brought out with a new and interesting clearness. The author concludes that the concept "spherical triangle" was introduced into mathematics by Menelaus, for although other Greek writers, including Autolycus, Euclid, Hypsicles, and Theodosius of Tripoli, had written upon the sphere, no trace of any trigonometric treatment is found in their writings. Indeed the theory of plane trigonometry may also be said to take its scientific origin in this same work, although of course the science was known before this time, notably by Hipparchus. But interesting as are all of these facts, far more so is the proof adduced to show that Menelaus recognized the projectivity of the anharmonic ratio on the sphere, and stated it as a fact known to his predecessors, probably including Hipparchus. Then with his characteristic thoroughness the author traces this proof through the Arab translators and back through the late Latin, to show how the fact happened to be lost for so many centuries.

The second part of the volume is devoted to certain "Nachträge und Berichtigungen zu 'Die Mathematiker und Astronomen der Araber und ihre Werke'" which appeared in volume 10 of the *Abhandlungen*.

The third part consists of an essay by Karl Bopp, "Antoine Arnauld, der grosse Arnauld, als Mathematiker." The idea has never been entirely wanting that Arnauld was a mathematician of power. Cantor mentions him, although only in relation to a controversy between Leibnitz and Guido Grandi; but even by his own countrymen his contributions to mathematics have long since been forgotten. Arnauld (b. 1612) lived in the time of Louis XIV, and received the name of "le grand Arnauld" because of his philosophical and theological attainments. Indeed it is in his philosophical writings that many of his mathematical ideas are suggested. It is, however, in his *Nouveaux Elémens de Géométrie* (1667) that he appears as a mathematician purely, and it is to this work that Herr Bopp devotes his chief attention. In Besoigne's *Histoire de l'abbaye de Port Royal* (1752) an anecdote of M. Nicole is given to the effect that Pascal having one day showed Arnauld a manuscript of his own on Euclid, the latter objected because the faulty arrangement of the Greek was unchanged. Thereupon Pascal challenged Arnauld to write a better work, a suggestion to which the latter acquiesced, and the "New Elements" was the result.

It is not worth while to enter into the general nature of the work. It was elementary but original. One or two of the most interesting features, however, deserve mention. The author uses in one place the expressions $\frac{1}{10}$ d'x, $\frac{1}{10}$ d'y, to represent these fractional parts of x and y, which leads Herr Bopp to suggest that this French symbolism might have suggested to Leibnitz his dx, an idea that is interesting even if far-fetched. He also goes more extensively into the theory of triangular numbers than had any of his predecessors, a fact which shows the influence of the Pascal-Fermat school and the overstepping of the traditional boundaries of geometry. Euclid's parallel postulate is passed with little question, for, he says, "elle a assez de clarté pour s'en contentes et ce seroit perdre de temps inutilement que de se rompre la tête pour la prouver par un long circuit," showing that his vision in this respect was no clearer than that of his contemporaries. He definitely asserts, however, the impossibility of solving the trisection problem by elementary geometry, "c'est à dire en n'y employant que des lignes droites et circulaires." Here, too, is found, five years before Pascal's publication, the latter's method of complete induction, a method which Maurolycus also understood long before either, but which was not generally appreciated. Such phrases as "it is necessary and sufficient" show that his thought was much ahead of the elementary writers of his period, while many of his proofs had a marked influence on the later French school. His generalization of propositions shed a new light on Euclid, and it is probably not too much to say that the French elementary geometry broke away from the Greek traditions largely through the influence of his initiative. The work closes with Arnauld's contributions to the theory of magic squares, a contribution that was original and powerful, and exceeded in value anything of the kind that had been attempted before his time.

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THERE is, so far as we know, no scientific annual which contains anything like the amount of information which we find in the Annuaire. It appears to be an attempt to satisfy the needs of everyone except, perhaps, the pure mathematician. It con-