of p which divides the order of a group H, and if no operator of H whose order is prime to p transforms a subgroup of order p^{μ} into itself without being commutative with each one of its operators, then must H contain a subgroup of index p^{λ} which is composed of all the operators of H whose orders are not divisible by p. Frobenius observes that it is possible to state this theorem somewhat more generally; but in this case the statement becomes still more complex and we shall not present it here.

STANFORD UNIVERSITY,
June, 1902.

SHORTER NOTICES.

Urkunden zur Geschichte der Mathematik im Mittelalter und der Renaissance. By M. CURTZE. Erster Theil. *Abhandlungen zur Geschichte der mathematischen Wissenschaften*, XII. Heft. Leipzig, Teubner, 1902. 336 pp. 16 Marks.

IT is a compliment to the monumental work of Professor M. Cantor that the activity in the field of the history of mathematics for the past twenty years has been almost entirely directed by him. The sole effort has been to supplement his work, to enter some of the innumerable doors which he has opened, to decipher the inscriptions upon the monument which he has erected. Hardly an article appears in the *Bibliotheca Mathematica*, relating to the period preceding the middle of the eighteenth century, that does not refer in some way to Cantor's work, and the Abhandlungen have been more or less under his direction for a quarter of a century.

Herr Curtze's latest contribution is an evidence in point, not merely in being dedicated to Professor Cantor on the occasion of his doctor's jubilee, but in that it elaborates certain details of his History for which elaboration scholars have been waiting.

Half of the work is given to the *Liber embadorum* of Abraham Savasorda (Sahib al Schorta, chief of the guards) as translated from the Hebrew by Plato of Tivoli in 1116, a treatise merely mentioned by Cantor.* The work has already been noticed by Curtze † as being one of the chief sources of

* Vorlesungen, vol. 2, p. 853.
† *Bibliotheca Mathematica*, vol. 1, 3d series (1900), p. 501.

Leonardo of Pisa's Practica Geometriæ, and Braunmühl has testified * to its importance in the history of trigonometry. Of course it has also been more or less known to other historians, particularly to Steinschneider † who has devoted so much attention to Jewish mathematics.‡ The work itself, however, has been closed to the world at large until the appearance of the present edition.

It speaks well for the brotherhood of nations, as also of scholars, that the French government should permit the two manuscripts, which form the basis for the present edition, to be sent from the National Library and be placed at the disposal of Herr Curtze. . As a result we have, on opposite pages, the Latin and German texts, the former with the variations in reading of the two manuscripts.

The most interesting fact relating to the treatise is that it shows at the same time the chief source of Fibonacci's work on geometry, and the originality of the latter. Both of these facts appear throughout the treatise, and are emphasized in the footnotes. While the work is devoted chiefly to geometry, it is also valuable as giving insight into the Spanish arithmetic of the period. It shows that, among the learned class, the subject was of the Boethian stamp, the Hindu numerals being unknown, and the definitions being mainly the conventional ones of the middle ages. *Mutabemini numeri* and *almugesem numeri* are new names, however, and the explanation of the meaning of the product of a line by a line shows a broadening of definition due to the Arab influence. Abraham solves certain equations, as $x^2 + b = ax$ (giving the double root), $x^2 + y^2 = a^2$, $x \pm y = b$, and $x^2 + y^2 = a^2$, $xy = b$, of course geometrically. The geometry, as was the custom of the time, due both to Arab and to Roman influence, is largely given to mensuration. The value of π is given as $3\frac{1}{7}$ or 377/120 (that of Ptolemy), and the treatment of the circle is followed by some discussion of trigonometry, the table of chords being probably the oldest that is known in Latin form.

The second part of the volume is devoted to the correspondence of Regiomontanus with Giovanni Bianchini, Jacob von Speier and Christian Roder. This correspondence is preserved in the city library at Nürnberg, and although once published,§

* Geschichte der Trigonometrie, vol. 1, p. 93.

† *Zeitschrift für Mathematik und Physik*, XII, 1.

‡ See also his articles in the *Bibliotheca Mathematica*.

§ Christophori Theophili de Murr Memorabilia Bibliothecarum Publicarum Norimbergensum et Universitatis Altorfinæ. Pars 1., M.DCC.LXXXVI.

its reappearance is valuable both because it corrects certain errors in Murr's transcription, and because it now becomes generally accessible.

Regiomontanus was the leader of his generation in astronomy and mathematics, and his correspondence with Bianchini, who was court astronomer to the Duke of Ferrara, Speier, who was court astrologer to the Prince of Urbino, and Roder, the professor of mathematics at the University of Erfurt, throws much light upon the practical astronomical work of the fifteenth century. The correspondence is in Latin and no translation is given.

Altogether, this number of the Abhandlungen is one of the most valuable that have appeared, and the tendency to publish the sources for the history of mathematics is one that will meet the hearty commendation of scholars.

DAVID EUGENE SMITH.

Gauss' wissenschaftliches Tagebuch, 1796–1814. Mit Anmerkungen herausgegeben von FELIX KLEIN. Reprinted from the *Festschrift zur Feier des hundertfünfzigjährigen Bestehens der Königlichen Gesellschaft der Wissenschaften zu Göttingen.* Berlin, Weidmannsche Buchhandlung, 1901, 8vo., 44 pp.

As a youth not quite nineteen years old Gauss began jotting down in a copy-book memoranda, always, unfortunately, of the very briefest sort, of the great mathematical discoveries he was making. The entries in this Scientific Diary (*Catalogus*, Gauss calls it) are in Latin, and begin with a statement dated March 30, 1796, to the effect that Gauss had found a construction for the regular polygon of seventeen sides. From this date the entries follow each other in rapid succession, there being no less than 112 in the next four years and a quarter. From here on they become more irregular, and there are only 34 entries during the following fourteen years. Such a diary as this, written by any great mathematician, would be of the greatest interest, as illustrating, even with all its gaps and obscurities, the order in which the mathematical ideas developed in his mind and the form they first took; but there is probably no mathematician in whose case it could be even approximately as valuable as in the case of Gauss. For it is well known that ideas, many of them of the first importance, poured in on Gauss's mind in his early youth in such numbers that, as he himself