$n_1+1, \cdots$, then the group generated by i_1 and H contains operators of order p^2 and the remarks in regard to additional groups apply only to the remaining numbers and to the invariant operators of H which are not commutators. As i_1 and its conjugates cannot give rise to any group of order p^m when p is less than some one of the numbers $n+1$, $n_1+1, \cdots$, all the groups of this order which contain H can be readily obtained by the above considerations. It may be observed that this includes all the groups of order p^m in which every operator is of order p whenever $m<5$, since every group of order p^4 contains an abelian subgroup of order p^3.

STANFORD UNIVERSITY,
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A CLASS OF SIMPLY TRANSITIVE LINEAR GROUPS.

BY PROFESSOR L. E. DICKSON.

1. IN the study of the group defined for any given field by the multiplication table of any given finite group,* it is necessary to discuss the types of simply transitive linear homogeneous groups G whose transformations can be given the form

$$(1)\qquad \begin{gathered} \xi_1'=\eta_1\xi_1,\quad \xi_2'=\eta_2\xi_1+\eta_1\xi_2,\quad \xi_3'=\eta_3\xi_1+\alpha\xi_2+\eta_1\xi_3,\\ \xi_4'=\eta_4\xi_1+\beta\xi_2+\gamma\xi_3+\eta_1\xi_4,\\ \xi_5'=\eta_5\xi_1+\lambda\xi_2+\mu\xi_3+\nu\xi_4+\eta_1\xi_5,\cdots. \end{gathered}$$

Here $\eta_1, \eta_2, \eta_3, \eta_4, \eta_5, \cdots$ are the independent parameters, while $\alpha, \beta, \gamma, \lambda, \cdots$ are linear homogeneous functions of the η_i. Burnside† was led to the erroneous conclusion that every such group G is an abelian group. He first concludes that the expression for ξ_t' contains only the parameters $\eta_1, \cdots, \eta_t$ and contains η_t only in the first term $\eta_t\xi_1$. That this result need not be true is shown by a consideration of the simply transitive group of quaternary transformations

$$(2)\qquad \begin{gathered} \xi_1'=\eta_1\xi_1,\quad \xi_2'=\eta_2\xi_1+\eta_1\xi_2,\quad \xi_3'=\eta_3\xi_1+\alpha\xi_2+\eta_1\xi_3,\\ \xi_4'=\eta_4\xi_1-\frac{a_3}{a_4}\alpha\xi_2+\eta_1\xi_4, \end{gathered}$$

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* For the case of a continuous field, Burnside, *Proc. Lond. Math. Soc.*, vol. 29 (1898), pp. 207–224, 546–565; for an arbitrary field, Dickson, *Transactions*, vol. 3 (1902), pp. 285–301.

† *Proc. Lond. Math. Soc.*, vol. 29, pp. 552–553.

where $a \equiv a_2\eta_2 + a_3\eta_3 + a_4\eta_4$, $a_4 \neq 0$. Let Y_i be the infinitesimal transformation obtained by setting $\eta_i = \delta t$, $\eta_j = 0$ $(j = 1, 2, 3, 4 ; j \neq i)$. Then

$$Y_1 = \sum_{i=1}^{4} \xi_i \frac{\partial f}{\partial \xi_1}, \quad Y_2 = \xi_1 \frac{\partial f}{\partial \xi_2} + a_2 \xi_2 \frac{\partial f}{\partial \xi_3} - \frac{a_3}{a_4} a_2 \xi_2 \frac{\partial f}{\partial \xi_4},$$

$$Y_3 = (\xi_1 + a_3 \xi_2) \frac{\partial f}{\partial \xi_3} - \frac{a_3^2}{a_4} \xi_2 \frac{\partial f}{\partial \xi_4},$$

$$Y_4 = a_4 \xi_2 \frac{\partial f}{\partial \xi_3} + (\xi_1 - a_3 \xi_2) \frac{\partial f}{\partial \xi_4}, \quad (Y_2 Y_3) = a_3 Y_3 - \frac{a_3^2}{a_4} Y_4,$$

$$(Y_2 Y_4) = a_4 Y_3 - a_3 Y_4, \quad (Y_3 Y_4) = 0, \quad (Y_1 Y_i) = 0.$$

But the desired result can always be reached by applying a suitable transformation on the variables ξ_i and the cogredient transformation on the parameters η_i (see §§ 4, 5 below). Taking G in this reduced form, Burnside attempts to prove by induction that G is abelian. He supposes that the first $t - 1$ equations of G define an abelian group and concludes, from the fact that G is its own parameter group, that the subgroup of G generated by the infinitesimal operations Y_1, $Y_2, \cdots, Y_{t-1}$ corresponding to $\eta_1, \eta_2, \cdots, \eta_{t-1}$ is abelian. The invalidity of the conclusion is shown by an example. The transformations

$$(3) \quad \begin{aligned} &\xi_1' = \eta_1 \xi_1, \quad \xi_2' = \eta_2 \xi_1 + \eta_1 \xi_2, \quad \xi_3' = \eta_3 \xi_1 + \eta_1 \xi_3, \\ &\xi_4' = \eta_4 \xi_1 + (b_2 \eta_2 + b_3 \eta_3)\, \xi_2 + (c_2 \eta_2 + c_3 \eta_3) \xi_3 + \eta_1 \xi_4 \end{aligned}$$

constitute a simply transitive group in reduced form, which is its own parameter group. The first three equations taken alone constitute an abelian group. But Y_1, Y_2, Y_3 do not generate a group if $b_3 \neq c_2$. In fact,

$$Y_1 = \sum_{i=1}^{4} \xi_i \frac{\partial f}{\partial \xi_i}, \quad Y_2 = \xi_1 \frac{\partial f}{\partial \xi_2} + (b_2 \xi_2 + c_2 \xi_3) \frac{\partial f}{\partial \xi_4},$$

$$Y_3 = \xi_1 \frac{\partial f}{\partial \xi_3} + (b_3 \xi_2 + c_3 \xi_3) \frac{\partial f}{\partial \xi_4}, \quad Y_4 = \xi_1 \frac{\partial f}{\partial \xi_4},$$

$$(Y_1 Y_2) = 0, \quad (Y_1 Y_3) = 0, \quad (Y_2 Y_3) = (b_3 - c_2) Y_4,$$

$$(Y_i Y_4) = 0 \qquad (i = 1, 2, 3).$$

Each of the two preceding examples shows that G need not be abelian.

2. For the case of one variable or the case of two variables,

the transformations (1) evidently form a simply transitive abelian group. We proceed to consider the cases $n = 3, 4, 5$. The method applies immediately to n variables, but the formulæ are complicated by the necessary use of a triple subscript notation for the coefficients. Set

$$\alpha = \sum_{k=1}^{n} a_k\eta_k, \quad \beta = \sum b_k\eta_k, \quad \gamma = \sum c_k\eta_k,$$
$$\lambda = \sum l_k\eta_k, \quad \mu = \sum m_k\eta_k, \quad \nu = \sum n_k\eta_k,$$

where the $a_k, \cdots, n_k$ are constants for a particular group G. The general transformation of G will be designated T_η or $T_{\eta_1, \cdots, \eta_n}$. Then the product $T_{\eta'}T_\eta$ is of the form (1) with η_i'', α'', β'', $\cdots$, in place of η_i, α, β, where

$$\eta_1'' = \eta_1\eta_1', \quad \eta_2'' = \eta_2\eta_1' + \eta_1\eta_2', \quad \eta_3'' = \eta_3\eta_1' + \alpha\eta_2' + \eta_1\eta_3',$$
$$\eta_4'' = \eta_4\eta_1' + \beta\eta_2' + \gamma\eta_3' + \eta_1\eta_4',$$
$$\eta_5'' = \eta_5\eta_1' + \lambda\eta_2' + \mu\eta_3' + \nu\eta_4' + \eta_1\eta_5',$$
$$\alpha'' = \alpha\eta_1' + \eta_1\alpha', \quad \beta'' = \beta\eta_1' + \gamma\alpha' + \eta_1\beta', \quad \gamma'' = \gamma\eta_1' + \gamma'\eta_1,$$
$$\lambda'' = \lambda\eta_1' + \mu\alpha' + \nu\beta' + \eta_1\lambda', \quad \mu'' = \mu\eta_1' + \nu\gamma' + \eta_1\mu',$$
$$\nu'' = \nu\eta_1' + \eta_1\nu'.$$

The transformations (1) will form a group if, and only if, $T_{\eta'}T_\eta = T_{\eta''}$, where $\eta_1'', \cdots, \eta_n''$ have the values just given, while the relations

$$\alpha'' = \sum_{k=1}^{n} a_k\eta_k'', \quad \beta'' = \sum_{k=1}^{n} b_k\eta_k'', \quad \cdots$$

reduce to identities in η_i and η_i'. Upon replacing α'', β'', $\cdots$, η_k'' by the above values. Comparing the coefficients of $\eta_1\eta_1'$, we find that a_1, b_1, c_1, l_1, m_1, n_1 are zero. For $n = 5$, the remaining conditions are

$$(4) \quad a_3a_k + a_4b_k + a_5l_k = 0, \quad a_4c_k + a_5m_k = 0, \quad a_5n_k = 0,$$

$$(5) \quad c_3a_k + c_4b_k + c_5l_k = 0, \quad c_4c_k + c_5m_k = 0, \quad c_5n_k = 0,$$

$$(6) \quad n_3a_k + n_4b_k + n_5l_k = 0, \quad n_4c_k + n_5m_k = 0, \quad n_5n_k = 0,$$

$$(7) \quad \left\{ \begin{array}{c} b_3a_k + b_4b_k + b_5l_k = a_2c_k, \quad b_4c_k + b_5m_k = a_3c_k, \\ b_5n_k = a_4c_k, \quad 0 = a_5c_k, \end{array} \right.$$

$$(8)\quad \begin{cases} m_3 a_k + m_4 b_k + m_5 l_k = c_2 n_k, & m_4 c_k + m_5 m_k = c_3 n_k, \\ m_5 n_k = c_4 n_k, & 0 = c_5 n_k, \end{cases}$$

$$(9)\quad \begin{cases} l_3 a_k + l_4 b_k + l_5 l_k = a_2 m_k + b_2 n_k, & l_4 c_k + l_5 m_k = a_3 m_k + b_3 n_k, \\ l_5 n_k = a_4 m_k + b_4 n_k, & 0 = a_5 m_k + b_5 n_k, \end{cases}$$

where k takes the values 2, 3, 4, 5.

3. For $n = 3$, the a_k are the only coefficients to be considered, and the preceding conditions reduce to $a_3 a_k = 0 (k = 2, 3)$. Hence $\alpha = a_2 \eta_2$. Then η_1'', η_2'', η_3'', α'' are symmetrical in η_i and η_i', so that the group is abelian.

4. For $n = 4$, l_k, m_k, n_k do not occur, so that conditions are

$$a_3 a_k + a_4 b_k = 0, \quad a_4 c_k = 0, \quad c_3 a_k + c_4 b_k = 0, \quad c_4 c_k = 0,$$
$$b_3 a_k + b_4 b_k = a_2 c_k, \quad b_4 c_k = a_3 c_k, \quad 0 = a_4 c_k,$$
$$(k = 2, 3, 4).$$

Hence $c_4 = 0$. If either c_2 or c_3 is not zero, the conditions reduce to

$$a_4 = 0, \quad b_4 = a_3 = 0, \quad c_3 a_2 = 0, \quad b_3 a_2 = a_2 c_2.$$

If $a_2 \neq 0$, then $\alpha = a_2 \eta_2$, $\beta = b_2 \eta_2 + b_3 \eta_3$, $\gamma = b_3 \eta_2$, and η_j'', α'', β'', γ'' are symmetrical in η_i and η_i'. The group is therefore abelian. If $a_2 = 0$, T_η takes the form (3). The group G is abelian if, and only if, $b_3 = c_2$. If* $b_3 \neq c_2$, the only "ausgezeichnete" infinitesimal transformations are the $e_1 Y_1 + e_4 Y_4$.

Let next $c_2 = c_3 = 0$, so that the conditions are

$$a_3 a_k + a_4 b_k = 0, \quad b_3 a_k + b_4 b_k = 0 \qquad (k = 2, 3, 4).$$

If $a_4 = 0$, then $a_3 = b_4 = 0$, $b_3 a_2 = 0$. If also $a_2 = 0$, T_η is of the form (3) with $c_2 = c_3 = 0$. But if $a_2 \neq 0$, then $\alpha = a_2 \eta_2$, $\beta = b_2 \eta_2$, $\gamma = 0$, so that the group is abelian. Finally, if $a_4 \neq 0$, the conditions reduce to the following:

$$a_3 + b_4 = 0, \quad a_3^2 + a_4 b_3 = 0, \quad a_3 a_2 + a_4 b_2 = 0,$$

whence $\beta = -\alpha a_3 / a_4$, $\gamma = 0$, so that T_η is of the form (2). The group G is then not abelian (§1). To bring it to the reduced form, set

* The group is of the type (V′), page 588, Lie-Scheffers, Continuierliche Gruppen.

$$x_3 = a_3\xi_3 + a_4\xi_4, \quad \zeta_3 = a_3\eta_3 + a_4\eta_4.$$

Then T_η becomes

$$\xi_1' = \eta_1\xi_1, \quad \xi_2' = \eta_2\xi_1 + \eta_1\xi_2, \quad x_3' = \zeta_3\xi_1 + \eta_1 x_3,$$

$$\xi_4' = \eta_4\xi_1 - \frac{a_3}{a_4}(a_2\eta_2 + \zeta_3)\xi_2 + \eta_1\xi_4.$$

Its self-conjugate transformations are the following :

$$\xi_1' = \eta_1'\xi_1, \quad \xi_2' = \eta_1'\xi_2, \quad x_3' = \eta_1' x_3, \quad \xi_4' = \eta_4'\xi_1 + \eta_1'\xi_4.$$

The group of transformations (1) *on four variables is either abelian or else is one of the types* (2) *and* (3), *whose self-conjugate transformations form groups of exactly two parameters.*

5. Let next $n = 5$. Then $n_5 = 0$. If $a_5 \neq 0$, the last of the conditions (4) and (7) give $n_k = 0$, $c_k = 0$, and the second condition (4) gives $m_k = 0$. Hence $\gamma = \mu = \nu = 0$. The first condition (4) gives $a_3\alpha + a_4\beta + a_5\lambda = 0$. Set

$$x_3 = a_3\xi_3 + a_4\xi_4 + a_5\xi_5, \quad \zeta_3 = a_3\eta_3 + a_4\eta_4 + a_5\eta_5.$$

Then $x_3' = \zeta_3\xi_1 + \eta_1 x_3$, so that, by applying a transformation on ξ_3, ξ_4, ξ_5 and a transformation on the parameters η_3, η_4, η_5, we obtain a transformation (1) with $\alpha = 0$.

Let $a_5 = 0$, $a_4 \neq 0$. Then $c_k = 0$, so that $\gamma = 0$, and $a_3\alpha + a_4\beta = 0$. Set

$$x_3 = a_3\xi_3 + a_4\xi_4, \quad \zeta_3 = a_3\eta_3 + a_4\eta_4.$$

Then $x_3' = \zeta_3\xi_1 + \eta_1 x_3$. If $a_5 = a_4 = 0$, then $a_3 a_k = 0$ by (4), so that $a_3 = 0$.

Let $a_5 = a_4 = a_3 = 0$, $c_5 \neq 0$. Then $n_k = 0$ by the third equation (5), so that $\nu = 0$. Also $c_3 a_2\eta_2 + c_4\beta + c_5\lambda = 0$, $c_4\gamma + c_5\mu = 0$ by the first and second equations (5). Set

$$x_4 = c_4\xi_4 + c_5\xi_5, \quad \zeta_4 = c_4\eta_4 + c_5\eta_5.$$

Then $x_4' = \zeta_4\xi_1 - c_3 a_2\eta_2\xi_2 + \eta_1 x_4$. Hence, by applying a transformation on ξ_4, ξ_5 and one on the parameters η_4, η_5, we obtain a transformation (1) with $\gamma = 0$.

Let $a_5 = a_4 = a_3 = 0$, $c_5 = 0$. Then $c_4 = 0$ by (5). If $b_5 \neq 0$, then $n_k = 0$ by the third equation (7), so that $\nu = 0$. By the first and second equations (7),

$$(b_3 - c_2)a_2\eta_2 - a_2 c_3\eta_3 + b_4\beta + b_5\lambda = 0, \quad b_4\gamma + b_5\mu = 0.$$

Hence

$$x_4' = \zeta_4\xi_1 + [(c_2 - b_3)a_2\eta_2 + a_2c_3\eta_3]\xi_2 + \eta_1 x_4,$$
$$x_4 \equiv b_4\xi_4 + b_5\xi_5,\ \zeta_4 \equiv b_4\eta_4 + b_5\eta_5.$$

We may therefore take $b_5 = 0$. Then $b_4 b_k = 0$ by the first equation (7), so that $b_4 = 0$. Then $m_5 = 0$ by the second equation (8). Hence $l_5 = 0$ by the first equation (9). Hence T_η becomes

$$\xi_1' = \eta_1\xi_1,\quad \xi_2' = \eta_2\xi_1 + \eta_1\xi_2,\quad \xi_3' = \eta_3\xi_1 + a_2\eta_2\xi_2 + \eta_1\xi_3,$$
$$\xi_4' = \eta_4\xi_1 + (b_2\eta_2 + b_3\eta_3)\,\xi_2 + (c_2\eta_2 + c_3\eta_3)\,\xi_3 + \eta_1\xi_4,$$
$$\xi_5' = \eta_5\xi_1 + (l_2\eta_2 + l_3\eta_3 + l_4\eta_4)\,\xi_2 + (m_2\eta_2 + m_3\eta_3 + m_4\eta_4)\,\xi_3$$
$$+ (n_2\eta_2 + n_3\eta_3 + n_4\eta_4)\,\xi_4 + \eta_1\xi_5.$$

Since the group is now in its reduced form, it contains the self-conjugate transformations, in which η_1' and η_5' are arbitrary, $\eta_1' \neq 0$,

$$\xi_i' = \eta_1'\xi_i\,(i = 1, 2, 3, 4),\quad \xi_5' = \eta_5'\xi_1 + \eta_1'\xi_5.$$

6. The conditions (4)–(9) on T_η in its reduced form are

$$c_3a_2 = 0,\quad n_3a_2 + n_4b_2 = 0,\quad n_4b_3 = 0,\quad n_4c_2 = 0,\quad n_4c_3 = 0,$$
$$b_3a_2 = a_2c_2,\quad m_3a_2 + m_4b_2 = c_2n_2,\quad m_4b_3 = c_2n_3,\quad m_4c_2 = c_3n_2,$$
$$m_4c_3 = c_3n_3,\quad l_3a_2 + l_4b_2 = a_2m_2 + b_2n_2,\quad l_4b_3 = a_2m_3 + b_2n_3,$$
$$0 = a_2m_4 + b_2n_4,\quad l_4c_2 = b_3n_2,\quad l_4c_3 = b_3n_3.$$

If $n_4 \neq 0$, then $b_3 = c_2 = c_3 = 0$, $n_3a_2 + n_4b_2 = 0$. Set

$$x_4 = n_3\xi_3 + n_4\xi_4,\quad \zeta_4 = n_3\eta_3 + n_4\eta_4.$$

Then $x_4' = \zeta_4\xi_1 + \eta_1x_4$. Hence by introducing x_4 in place of ξ_4, and ζ_4 in place of η_4, T_η retains its reduced form and has $b_2 = b_3 = c_2 = c_3 = 0$. Then

$$(10)\qquad n_3a_2 = 0,\quad m_3a_2 = 0,\quad l_3a_2 = m_2a_2,\quad m_4a_2 = 0,$$

are the only further conditions.

If $a_2 \neq 0$, we obtain the transformation

$$\xi_1' = \eta_1\xi_1,\quad \xi_2' = \eta_2\xi_1 + \eta_1\xi_2,\quad \xi_3' = \eta_3\xi_1 + a_2\eta_2\xi_2 + \eta_1\xi_3,$$
$$\xi_4' = \eta_4\xi_1 + \eta_1\xi_4,$$
$$\xi_5' = \eta_5\xi_1 + (l_2\eta_2 + l_3\eta_3 + l_4\eta_4)\xi_2 + l_3\eta_2\xi_3 + (n_2\eta_2 + n_4\eta_4)\xi_4 + \eta_1\xi_5.$$

It is readily verified that these transformations form a group with the parameters $\eta_1 \cdots, \eta_5$, whatever be the values of $a_2, l_2, l_3, l_4, n_2, n_4$. The expressions for $\eta_i''(i = 1, 2, 3, 4)$ are symmetric in η_i and η_i', but that for η_5'' is symmetric if, and only if, $n_2 = l_4$. In the latter case only, the group is abelian. For $n_2 \neq l_4$, a transformation will belong also to the reciprocal group if, and only if, $\eta_2 = \eta_4 = 0$. Hence the subgroup of self-conjugate transformations has three arbitrary parameters η_1, η_3, η_5.

But, if $a_2 = 0$, the conditions (10) become identities. Hence the transformations

$$T_\eta \text{ with } a_2 = b_2 = b_3 = c_2 = c_3 = 0, \quad n_4 \neq 0,$$

form a group, whatever be the values of l_i, m_i, n_i. It is abelian if, and only if, $m_2 = l_3, n_2 = l_4, n_3 = m_4$. A self-conjugate transformation must have

$$\eta_2 = \eta_3 = 0, \quad \text{if } m_2 \neq l_3; \quad \eta_2 = \eta_4 = 0, \text{ if } n_2 \neq l_4;$$
$$\eta_3 = \eta_4 = 0, \quad \text{if } n_3 \neq m_4.$$

If G is not abelian, the subgroup of its self-conjugate transformations has two or three arbitrary parameters.

Let next $n_4 = 0$. If $a_2 \neq 0$, the conditions are

$$c_3 = n_3 = m_4 = 0, \quad b_3 = c_2, \quad (l_3 - m_2)a_2 = (n_2 - l_4)b_2,$$
$$m_3 a_2 = c_2 n_2 = l_4 c_2.$$

If also $n_2 = l_4$, so that $l_3 = m_2$, then $\eta_j''(j = 1, \cdots, 5)$ is symmetric in η_i and η_i' and the group is abelian. If $n_2 \neq l_4$, and $l_3 = m_2$, then $c_2 = 0$, $b_2 = 0$, $m_3 = 0$, so that

$$\xi_4' = \eta_4\xi_1 + \eta_1\xi_4, \; \xi_5' = \eta_5\xi_1 + (l_2\eta_2 + l_3\eta_3 + l_4\eta_4)\xi_2$$
$$+ l_3\eta_2\eta_3 + n_2\eta_2\xi_4 + \eta_1\xi_5,$$

with the restrictions $a_2 \neq 0$, $n_2 \neq l_4$. The self-conjugate transformations have $\eta_2 = \eta_4 = 0$, η_1, η_3, η_5 arbitrary. If $n_2 \neq l_4$, and $l_3 \neq m_3$, then $c_2 = m_3 = 0$, and the self-conjugate transformations have $\eta_2 = \eta_3 = \eta_4 = 0$, η_1 and η_3 arbitrary.

Let next $n_4 = a_2 = 0$. The conditions are

$$m_4 b_2 = c_2 n_2, \;\; m_4 b_3 = c_2 n_3, \;\; m_4 c_2 = c_3 n_2, \;\; (m_4 - n_3)c_3 = 0,$$
$$l_4 b_3 = b_2 n_3, \;\; l_4 c_2 = b_3 n_2, \;\; l_4 c_3 = b_3 n_3, \;\; (l_4 - n_2) b_2 = 0.$$

The transformations form an abelian group if, and only if

$$b_3 = c_2, \;\; l_3 = m_2, \;\; l_4 = n_2, \;\; m_4 = n_3, \;\; n_3 c_2 = c_3 n_2, \;\; n_3 b_2 = c_2 n_2.$$

The form of the general transformation can be simplified by applying a transformation on ξ_2, ξ_3, and the cogredient transformation on η_2, η_3, and similarly a transformation on ξ_4, ξ_5 and one on η_4, η_5.

7. The argument of Burnside, l. c., §6, page 553, is faulty. It does not show that $\nu = \mu$, but does prove that ν is a multiple of μ. In view of the work of Frobenius and that of Molien, the theorem in question is true.

THE UNIVERSITY OF CHICAGO,
May 12, 1902.

ERRORS IN LEGENDRE'S TABLES OF LINEAR DIVISORS.

BY DR. D. N. LEHMER.

SOME years ago an error in Legendre's Tables of Linear Forms came to my notice. Another was found recently by members of my class, and as this error was left without correction in the later editions I determined to make a careful computation of the whole set. I was surprised to find the list of errors so long. The importance of these tables for many investigations makes it desirable that all these corrections be noted. I have also compared results with the tables in Tshebyshef's Theorie der Congruenzen, Berlin, 1889. Most of the errors in Legendre's work have been carried over uncorrected into these tables.

I. Under the form $t^2 - 29u^2$ the form $116x + 3$ should read $116x + 7$. This error was corrected in the fourth edition (1900), which is a copy of the edition of 1830.

II. Under the form $t^2 - 38u^2$ the form $152x + 129$ should read $152x + 131$. Not corrected in the fourth edition nor in Tshebyshef.

III. Under the form $t^2 - 43u^2$ the form $172x + 147$ should read $172x + 137$. Not corrected in the fourth edition nor in Tshebyshef.

IV. Under $t^2 - 51u^2$ there are two forms $204x + 13$. The second of these should read $204x + 31$. This error is in the fourth edition but not in the first (1797).

V. Under $t^2 - 61u^2$ there are so many errors that I will give the correct list: $244x + 1$, 3, 5, 9, 13, 15, 19, 25, 27, 39, 41, 45, 47, 49, 57, 65, 73, 75, 77, 81, 83, 95, 97, 103, 107, 109, 113, 117, 119, 121, 123, 125, 127, 131, 135, 137, 141, 147, 149, 161, 163, 167, 169, 171, 179, 187, 195, 197, 199, 203, 205, 217, 219, 225, 229, 231, 235, 239, 241, 243. The