

corresponding multipliers are $a_1 a_{\frac{3a-2}{2}} \cdot a_{\beta-1} a_\beta$ and $a_2 a_{\frac{3a+4}{2}}$ respectively. By using the s_2 thus obtained and the given s_1 , we can therefore generate any alternating or symmetric group whose degree exceeds 9 and is divisible by 3. When $n = \beta + 1$ and a is odd, we may multiply s_2' by $a_1 a_{\frac{3a+1}{2}} \cdot a_\beta a_{\beta+1}$ and $a_1 a_{\frac{3a-5}{2}} \cdot a_\beta a_{\beta+1} \cdot a_{\beta-4} a_{\beta-1}$ respectively to obtain the required s_2 ; while the corresponding multipliers are $a_1 a_{\frac{3a-2}{2}} \cdot a_{\beta-4} a_{\beta+1} \cdot a_{\beta-1} a_\beta$ and $a_1 a_{\beta+1} \cdot a_2 a_{\frac{3a+4}{2}}$ when a is even.* Finally, when $n = \beta + 2$ and a is odd, suitable factors of s_2' are $a_1 a_{\frac{3a+1}{2}} \cdot a_{\beta-1} a_{\beta+1} \cdot a_\beta a_{\beta+2}$ and $a_1 a_{\frac{3a-5}{2}} \cdot a_{\beta-7} a_{\beta-4} \cdot a_{\beta-1} a_{\beta+1} \cdot a_\beta a_{\beta+2}$; when a is even the factors are $a_1 a_{\frac{3a-2}{2}} \cdot a_{\beta-7} a_{\beta+2} \cdot a_{\beta-4} a_{\beta+1} \cdot a_{\beta-1} a_\beta$ and $a_1 a_{\beta+1} \cdot a_2 a_{\frac{3a+4}{2}} \cdot a_\beta a_{\beta+2}$, respectively.

When $n < 12$, it is easy to examine the cases directly and thus prove that *every alternating group with the exception of those of degrees 3, 6, 7, 8 is generated by two of its substitutions of orders two and three respectively, and every symmetric group with the exception of those of degrees 5, 6, 8 is generated by two of its substitutions of the same orders.* Hence 6 and 8 are the only degrees for which neither the symmetric nor the alternating group is generated by two of its substitutions of orders two and three respectively. The only transitive groups generated in these two cases are of orders 6, 12, 18, 24, 48, 60, 168 and 336.

A CURIOUS APPROXIMATE CONSTRUCTION FOR π .

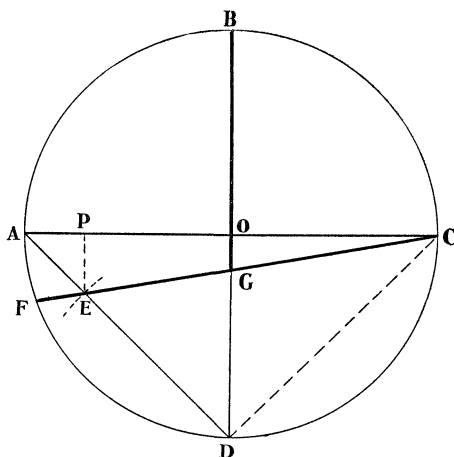
BY MR. GEORGE PEIRCE.

(Read before the American Mathematical Society, April 27, 1901.)

CONSTRUCTION: AC, BD are perpendicular diameters of a circle. $DE = AO$. CE produced cuts BD in G and the circle in F .

To prove that $\pi r = FC + BG$ approximately.

* When $n = 13$ we may assume $a = 2$, *Quar. Jour. of Math.*, vol. 29 (1893), p. 228.



Draw EP perpendicular to AC . Then $CE = r\sqrt{3}$. Also

$$FE = AE \cdot ED / EC = r(\sqrt{2} - 1) / \sqrt{3}.$$

Therefore

$$FC = CE + FE = \frac{1}{3}r(\sqrt{6} + 2\sqrt{3}).$$

Again

$$OG = PE \cdot OC / PC = PE \cdot OC / \sqrt{EC^2 - PE^2}$$

$$= r(1 - \frac{1}{2}\sqrt{2}) / \sqrt{3 - (1 - \frac{1}{2}\sqrt{2})^2} = r(3 - 2\sqrt{2}),$$

and

$$BG = BO + OG = r(4 - 2\sqrt{2})$$

Hence

$$FC + BG = r(4 - 2\sqrt{2} + \frac{2}{3}\sqrt{3} + \frac{1}{3}\sqrt{6}) = 3.14277r.$$

Now $\pi = 3.14159$. Hence the error is less than $1/2600$ of the whole; in a circle of 75 yards radius the error would be about one inch.

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