

following each other under equal angles  $2/\pi n$ . In symbolic notation, advance or rotation through an angle  $2\pi/n$  is represented by multiplying by the quantity  $\cos 2\pi/n + j \sin 2\pi/n$ , and so the E.M.F.'s of a polyphase system are

$$E, E \left( \cos \frac{2\pi}{n} + j \sin \frac{2\pi}{n} \right), E \left( \cos \frac{4\pi}{n} + j \sin \frac{2\pi}{n} \right), \text{ etc.}$$

In chapters XXVI and XXVIII the author handles this application for the deduction of the expression for the rotating magnetic field, the ring and star E.M.F.'s of interlinked systems, and other matters of general use, but up to this time wanting an analytical expression.

The writer has only gratitude to express at the appearance of this work, and his one regret is that its author did not also include in it his recent articles on the rotary converter.

JOHN B. WHITEHEAD, JR.

JOHNS HOPKINS UNIVERSITY,  
April 18, 1901.

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#### SHORTER NOTICE.

*Leçons Nouvelles sur les Applications Géométriques du Calcul Différentiel.* By W. DE TANNENBERG. Paris, A. Hermann, 1899. 192 pp.

THIS volume, which M. de Tannenberg has contributed to the literature of the theory of curves and surfaces, is very opportune. We have wanted a book which would make possible for the beginner a knowledge of the more fundamental geometrical applications of the calculus and in a way which would prepare him for the treatises of Darboux and Bianchi. To be sure, this field has been covered, more or less, in the chapters devoted to geometrical applications in the French treatises on analysis—notably by Jordan, Picard, Appell—but rather as examples of the methods of analysis and not standing forth as a systematic development of the elements of another field of mathematics. Again, there have been in recent years, quite a number of shorter treatises with just the scope of the volume under discussion, but their treatment of the subject has been along lines quite different from the well known methods of the calculus: Ricci in his *Lezioni* arrives at the results by the study of

differential forms, Méray makes use of the methods which he has developed in his *Leçons nouvelles sur l'analyse infinitésimale*, Fehr adopts the vector method of Grassman.

But M. de Tannenberg has given us a treatment of the subject by methods which involve only the simplest notions of the calculus, and in a way that is at the same time clear, concise, and rigorous. Limiting himself to the study of curves and surfaces in the region of ordinary points, he considers the cartesian coördinates as developed according to ascending powers of one or two variables, and from the study of these developments determines many of the geometrical properties of the given locus. As one reads, he is impressed with the simplicity of the methods; very often an important result follows as a corollary, or is embodied in a remark, and at times the examples which are scattered through the book serve the double purpose of illustration and a step toward the further development of the subject. Very little is said about plane curves; and, when treated, it is simply as a particular case of curves in general.

The book has two main divisions: the first, consisting of two parts, treats of the descriptive properties of curves and surfaces; the second, in three parts, treats of their metric properties.

Of particular interest in the first division is the treatment of the theory of envelopes, whether of curves or surfaces. The conditions necessary and sufficient for the existence of envelopes are set forth so clearly that the beginner should not experience the usual difficulty with this subject. The second part closes with a brief discussion of systems of lines—ruled surfaces, congruences, and linear complexes.

In the third part of the book and the first of the second division, the notion of the geometrical derivative of a segment is introduced and is of great service in the definition of first curvature and torsion of a curve, and the derivation of the Serret formulæ. As an application of these formulas, the variation of a segment of a right line is discussed, and the results thus obtained enable the author to simplify the proofs of many theorems in the theory of surfaces. In particular, they are applied to the study of skew and developable surfaces; this forms the fourth part.

The fifth part is the most important; it comprises nearly one-half of the book and is devoted to the theory of curved surfaces. First the six fundamental functions, characteristic of a surface, are defined and the relations which they satisfy are found; then the general properties of curves traced on surfaces are discussed. From the theorems of

Meusnier and Bonnet follow the definitions of asymptotic lines and lines of curvature; their properties, together with those of conjugate lines, are then discussed and geodesic lines are defined. After an application of the preceding theories to several of the simpler surfaces—cones, surfaces of revolution, quadrics, developables, surfaces of Monge—the book closes with a treatment of the general equation of geodesic lines.

The print is good, the figures are helpful and the general arrangement is very attractive.

L. P. EISENHART.

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### NOTES.

THE secretaryship of Section A of the American association for the advancement of science has been filled by the appointment of Dr. G. A. MILLER. Papers intended for the programme of the Denver meeting (August 24–31) of Section A, should be sent to Dr. Miller, at 115 Cook St., Ithaca, N. Y.

THE April number of the *American Journal of Mathematics* (volume 23, number 2) contains the following articles: "The cross-ratio group of 120 quadratic Cremona transformations of the plane," by H. E. SLAUGHT; "Memoir on the algebra of symbolic logic," by A. N. WHITEHEAD; "On a special form of annular surfaces," by V. SNYDER; "On the transitive substitution groups whose order is a power of a prime number," by G. A. MILLER; "Geometry on the cubic scroll of the second kind," by F. C. FERRY.

AT the regular meeting of the London mathematical society held on May 9, 1901, the following papers were read: "A case of algebraic partitionment," by Major P. A. MACMAHON; "On the series whose terms are the cubes and higher powers of the binomial coefficients," by Major P. A. MACMAHON; "A property of recurring series," by Dr. G. B. MATHEWS; "The product of two spherical surface harmonic functions," by Mr. J. B. DALE.

THE German Society for the promotion of instruction in mathematics and science held its tenth annual meeting the last week of May at Giessen under the presidency of Dr. Rausch. The society has nine hundred members.

THE committee elected to judge the competition in 1901 for the Francoeur and Poncelet prizes of the Paris academy of sciences consists of Professors C. JORDAN, H. POINCARÉ, E. PICARD, P. APPELL, and MAURICE LEVY.