

ADAMS'S UNPUBLISHED PAPERS.

The Scientific Papers of John Couch Adams, Vol. II. Part I., *Extracts from Unpublished Manuscripts*, edited by R. A. SAMPSON. Part II., *Terrestrial Magnetism*, edited by W. G. ADAMS. Cambridge, Pitt Press, 1900. 4to. xxxii + 646 pp., with 6 magnetic charts.

Lectures on the Lunar Theory. By JOHN COUCH ADAMS, edited by R. A. SAMPSON. Cambridge, Pitt Press, 1900. 8vo. 88 pp.

By those who knew John Couch Adams best the examination of the manuscripts which he left and the publication of a selection from them has been looked forward to with much interest. His well known reluctance to publish anything not in a complete form—"I have some finishing touches to put to it"—raised hopes that valuable results might be contained in the packets of neatly-written and dated papers which were turned over after his death for examination. It was known that he had been at work on various problems, in particular on that most difficult one—the theory of Jupiter's satellites, and on the theory of terrestrial magnetism. It was known, too, that Adams rarely attacked a problem without throwing a new light upon it or evolving some unexpected result. He had not fallen into the modern habit of making "preliminary communications," or, at least, if these were made they were verbal and did not find their way into print. Thus the publication of this volume was looked forward to with greater expectation than is usual in the case of other scientists who have lived to his age.

I shall not be misunderstood in saying that a slight feeling of disappointment arises on turning over its pages and seeing the matter contained therein. Adams's reputation as a mathematician and astronomer of the highest class rests on too firm a basis to be disturbed by anything which may or may not be contained in his unpublished papers. His published work, small in quantity though it may be, made an ineffaceable mark on the subjects which he touched. The discovery of Neptune, the determination of the accurate value of the secular acceleration of the moon's mean motion, the method by which he arrived at the correct period of the November meteors are sufficient to stamp the character of his work without mentioning other results which show equal

ability but are not so well known. If we are disappointed at finding no great discovery in the new volume or anything indicating that he was on the track of one, we have no reason to regret its appearance or to feel that the labor and thought which the editors have expended in putting the manuscripts into form have been in any sense wasted. There is, in fact, almost nothing which is not new. Some of the papers consist of new methods in handling old problems, others consist of the detailed work in the solution of problems, the mere results of which had alone been published, and, more important than all these, his great work on terrestrial magnetism is set forth in a complete and connected form.

The first number consists of a selection from Adams's lectures on the lunar theory. These have also been published separately without change for the use of those to whom the larger volume might not be easily accessible. The editor, Professor R. A. Sampson, has not aimed at giving the lectures in the complete form in which Adams delivered them. Portions which contained alternative proofs or explanations which are to be found in the text books have been omitted. The result is a continuous development which gives the coefficients of the principal inequalities due to the sun's action with a good degree of approximation. In his preface, Professor Sampson says "Of current elementary theories it may be said that they leave off where the difficulties of the subject begin, that is to say, where the various cases of slow convergence have been exposed, but not dealt with. It is perhaps not too much to say that these lectures carry us to the point where such difficulties end, in an adequate evaluation of all the chief constants. They leave the problem effectively solved and not merely stated, and show the path clear for the formation of a detailed theory, if that is desired." This is hardly correct as to the question of convergence, or at any rate it obscures the issue. The difficulty of slow convergence can hardly be said to have been dealt with by Adams any more than by his predecessors. That he has obtained close approximations is due to the fact that the numerical value of m ,* the ratio of the mean motions, is used instead of series proceeding in powers of m . There is not much difficulty about convergence in the case of the moon when the numerical value of m is used at the outset, since slow convergence only occurs along powers of m . Again, one of the chief difficulties of the

* More accurately, he uses $m/(1 - m)$.

subject is the calculation of the higher terms with sufficient accuracy. Owing to small divisors the earlier terms have to be calculated with far greater accuracy than would be necessary if these small divisors did not arise, and this difficulty has not been bridged in any other way than by calculating the earlier terms to a high degree of accuracy.

The method used by Adams is practically the same as that of G. W. Hill, except that he has adopted polar where Hill uses rectangular coördinates. For the earlier inequalities this method works well enough, but it would most probably give rise to unmanageable calculations for the higher ones if a complete theory were in contemplation. In general, it lacks the elegance and capacity for high approximation with a given amount of labor which characterizes Hill's methods, with one or two exceptions, notably that by which Adams arrives at the equation from which the motion of the perigee is obtained. In fact, the symmetry possible with rectangular coördinates disappears altogether when polar coördinates are used, and in the latter case the expansions of such expressions as

$$\cos (a_0 + a_1 \cos t + a_2 \cos 2t + \dots)$$

have to be continually performed while, in the former, we have only to do with multiplications of algebraic series.

The next paper contains the detailed work necessary for the expression of the infinite determinant giving the motions of the node or perigee. In No. 3, entitled "Numerical developments in the lunar theory," we have the numerical values of the coefficients of δn when we put $n + \delta n$ for n and $n' + \delta n'$ for n' with $\delta n = \delta n'$ in the variational inequalities. The values of the coefficients of the parallactic inequalities are also obtained to 16 places of decimals, the first power of the ratio of the parallaxes only being retained. The determination of the part of the secular acceleration of the moon's mean motion which depends on m only is found in the next paper, and in No. 15 without having recourse to expansions in powers of m . Adams's method is in reality a numerical one and depends on the changes in the values of the coefficients of several periodic terms, the principal ones being those found in the previous paper. The writer has shown how this part of the secular acceleration can be found from the *constant* term in the parallax with any degree of accuracy required.*

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From these papers it appears that Adams had calculated the following sets of inequalities with the accuracy necessary for a complete lunar theory: The parts of the variational terms which depend on m only and those which depend on m and the second power of the solar eccentricity; the terms depending on m and the first powers of the solar eccentricity, the ratio of the parallaxes, and the inclination; and the principal part of the motion of the node. Other inequalities, *e. g.*, the terms depending on m and the first power of the lunar eccentricity had been found to a less degree of accuracy; some of these appear in the lectures.

A short note on Neison's lunar inequality due to Jupiter follows. Adams finds the theoretical coefficient of this with some accuracy by considering it as a sort of Jovian evection, thus making use of the literal expression for the solar evection. The value obtained differs by about 15 per cent. from that of M. Radau,* who has calculated a large number of the planetary inequalities.

No. 6 contains a novel suggestion for the solution of the equation

$$\frac{d^2w}{dt^2} + (a_0 + a_1 \cos 2t + a_2 \cos 4t + \dots)w = 0$$

which occurs so frequently in celestial mechanics. The solution is of the form

$$w = \sum A_i \cos \{(k + 2i)t + \alpha\},$$

and the difficulty of calculation arises from the presence of small divisors due to the fact that k is in general very nearly equal to unity. Adams's method really amounts to finding all the other coefficients in terms of k , A_{-1} in such a way that no small divisors occur; the two equations for these quantities are solved as a final step.

Papers 7, 8, 9, 12 are on Jupiter's satellites. As Professor Sampson has presented Adams's investigations, it appears that a new method of treating the whole subject was in contemplation, somewhat analogous to that which he used for the motion of the moon. He first solves the problem of the motion of a satellite about an oblate primary, in fact he finds the inequalities due to the figure of Jupiter. The equations for the three coördinates are reduced to two simultaneous ones of the second order with the distance and perpendicular on the equatorial plane as dependent vari-

* *Ann. de l' Observ. de Paris*, vol. 21.

ables and the time as independent variable. The equations are solved literally so that they can be applied to any satellite. The third coördinate is then easily found, as the editor shows in a short note. The development of

$$[a^2 - 2aa' \cos (nt - n't + \varepsilon - \varepsilon') + a'^2]^{-\frac{1}{2}},$$

necessary to obtain the mutual actions of the satellites on one another, is then made with some completeness and applied, for illustration, to the first two satellites. The mutual perturbations of two satellites when the eccentricities and inclinations are neglected next follow. Here again polar coördinates are used so that the perturbations are found directly, instead of indirectly by the more usual method of the variation of arbitrary constants. Values for the radius vector and the longitude, of the proper form, are substituted in the differential equations and the coefficients are determined by equations of condition.

As Adams does not go beyond the first order of the disturbing forces in the published investigations, the difficulty arising from the exact relation

$$n - 3n' + 2n'' = 0,$$

where n , n' , n'' are the mean motions of the first three satellites, does not come into consideration. In fact, as far as the theory is given, we have simply those inequalities which are analogous to the variational and parallactic terms in the lunar theory. Some numerical results are given. It would be interesting to see how this method of procedure would work out when applied to attack the complete problem. Perhaps Adams did not see his way clear for this and therefore refrained from publishing what he had already done. The chief difficulties undoubtedly arise mainly in the terms depending on the eccentricities and on the square of the disturbing forces owing chiefly to the smallness of $n - 2n'$, $n' - 2n''$ and to the exact equality of these two expressions. The other papers on this subject contain corrections of the masses of the satellites and of errors in Damoiseau's and Laplace's theories.

Every astronomer knows of H. A. Newton's investigations on the orbit of the November meteors in which he arrived at the conclusion that the observations could in general be satisfied by any one of five competing values for the mean period. Adams supplemented this by showing that only one of these five values was consistent with the observed motion of the node of the orbit. In paper 10 we have the

details of the method by which he arrived at the result. The brevity of the work is remarkable. The calculations are not given in detail. They are reduced to mechanical quadratures and a few of the principal steps for the perturbations by Jupiter are set forth. The method is a modification of Gauss's formulæ.

Some extracts from Adams's lectures on the Figure of the Earth occupy No. 11. They consist of theorems on the attractions of spheres and spheroids when the law of force is any function of the distance and conclude with the well-known differential equation connecting the ellipticity of strata with the density. A couple of notes on the analytical interpretation of Newton's methods for finding the variation and motion of the apse, a short method for obtaining certain of Laplace's expansions of functions in power series, and lists of errata in the works of Plana, Damoiseau, and de Pontécoulant occupy the remaining pages of Part I. In the last there are one or two slight errors. On p. 236, line 3, for 1025, read 1024; omit line 5, which is given on the last line but one of the previous page; on line 7 insert a + before the last number.

The second part, containing Adams's work on terrestrial magnetism, occupies two-thirds of the volume. The difficulties of Professor W. G. Adams, who undertook to edit this portion of the work left by his brother, must have been sufficiently great. The matter had not been put into final form, though it was probably ready for it. The editor's chief labor was to gather together the various packets of theoretical and numerical work; to discover their connection, and by blending them to set forth in a complete manner the results of Adams's labors. A glance at the memoir shows that this task can have been no light one, and students will be grateful to Professor W. G. Adams for the manner in which he has performed it. In a preface he gives a summary which shows well the nature of the work and the conclusions reached by his brother.

The method adopted is that of Gauss, with modifications. But instead of the 24 constants with which Gauss contented himself, Adams has included 240, just ten times as many. Moreover, the observations which in Gauss's time were few and badly distributed, have increased much in volume, though still far from what is really necessary for a thorough determination. The memoir is arranged in eight sections, of which the first four consist of various relations between the harmonics and coefficients used in such investigations. Sections V, VI, respectively treat of the theory of

terrestrial magnetism when the earth is considered as a sphere and as a spheroid. The last two sections contain the numerical calculations. At the end are maps drawn from the numerical results for every 5° of latitude and every 10° of longitude. The agreement with observation is stated by the editor to be very satisfactory.

In conclusion, the reader will gather from the rough indications here given that the volume is not a mere compilation of incomplete fragments. Whatever he may think about the value of collections of published papers, and in particular of Volume I of Adams's works, it will be immediately apparent that Volume II is at least on the same footing as any other book containing original and previously unpublished scientific work, and its possession by scientific libraries will be as much a necessity as is that of the best known journals and treatises.

ERNEST W. BROWN.

NOTICE SUR M. HERMITE.

PAR M. C. JORDAN.

ADDRESS DELIVERED AT THE MEETING OF THE PARIS ACADEMY OF SCIENCES, JANUARY 21, 1901.

THE French school of mathematics loses in the person of M. Hermite its head and master.

It would be rash to undertake to analyze in haste and under the stress of keen emotion the long series of his works which have thrown so much lustre on the second half of the nineteenth century. Such an undertaking calls for more time and calmer feelings. Addressing then to our venerated confrère the last farewell, which his modesty forbade pronouncing at his grave, we limit ourselves here to pointing out, in broad lines and as far as memory permits, some of the discoveries which we owe to him.

In 1843, M. Hermite entered the Ecole Polytechnique at the age of twenty years. At the suggestion of Liouville, he wrote to Jacobi communicating the results which he had obtained relative to the division of abelian functions, then but little known. The illustrious German geometer, who was occupied at the time with the editing of his works, did not hesitate to give the letter of his young correspondent a place beside his own investigations. He wrote to him a little later: "Do not be troubled, Monsieur, if some of