

SHORTER NOTICES.

Application de la Méthode Vectorielle de Grassmann à la Géométrie Infinitésimale. Par HENRI FEHR.* Georges Carré et C. Naud, Éditeurs. Paris, 1899. 94 pp.

THE mathematicians Grassmann and Hamilton who almost simultaneously published their first work upon a "calculus of space" were undoubtedly geometers, not analysts—abstract and speculative they may have been—but still geometers. We have only to regret that they were not also clear stylists; for it must be admitted that, with the exception of the attempts of certain persons to show how everything may be done by the methods of vector analysis or quaternions, nothing could be more fatal to the popular acceptance and use of a space analysis than the form in which it was presented to the public by the inventors. Succeeding writers for the most part seem to have erred along the same lines or to have forgotten the stress originally laid upon the interpretation of the analysis. It is therefore with great pleasure that we read M. Fehr's little book, which is written with such admirable clearness and selected with such tasteful care that in the compass of ninety-one pages there is included, without the slightest suspicion of crowding, a preface, an introduction on the use of vector analysis, and a fairly complete treatment of differential geometry.

M. Fehr originally wrote his book as a thesis to be presented for the degree of doctor of science at the University of Geneva. As a thesis, the work contains nothing original either in vector analysis or in geometry. All the results and methods were known well enough before. Indeed, anyone who has heard such lectures as are given at our leading universities upon these two branches of mathematics ought to be able to put together the material in this book with almost no difficulty. But to put it together in so pleasing a manner is a far harder task. The reviewer, therefore, must deal with the manner of presentation rather than with that which is presented.

The votaries of vector analysis will find in M. Fehr's book little to cheer them. In fact, so meager is the development of the subject that one might almost say the entire analysis consists of three symbols

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$$\begin{aligned}
 [a_1 a_2] &= V a_1 a_2, \text{ the vector product of two vectors,} \\
 [a_1 | a_2] &= -S a_1 a_2, \text{ the scalar product of two vectors,} \\
 [a_1 a_2 a_3] &= -S a_1 a_2 a_3, \text{ the scalar product of three vectors.}
 \end{aligned}$$

These expressions are used, to all intents and purposes, as abridged notations. If two vectors a_1 and a_2 are perpendicular, the fact is indicated by writing $[a_1 | a_2] = 0$; if they are parallel, by $[a_1 a_2] = 0$; and if three vectors a_1, a_2, a_3 are coplanar, by $[a_1 a_2, a_3] = 0$. The working rules of the analysis and the formal transformations of the symbols are developed scarcely at all. M. Fehr knows very well what to omit, and he omits a great deal. The differentiating operator ∇ , which can be used with considerable advantage in some portions of differential geometry, is not introduced at all. This omission tends to clearness and simplicity. The linear vector function is also left out to advantage. Indeed, it seems as if the formal methods of vector analysis are none too well suited to the development of differential geometry. The vectors appear more or less artificial. They are not thrust upon us irresistibly, as in physics. The need of a vector analysis is not so apparent as it might be. M. Fehr, therefore, has done well to introduce as little as possible of the vector machinery. Perhaps it is well to note that M. Appell has followed a similar course in his treatment of the theory of vectors: *Traité de mécanique rationnelle*, volume I, chapter I.

The presentation of the elements of differential geometry given by M. Fehr is hardly to be excelled. The usual important and fundamental results are obtained in much less space and time than is customary, and yet with perhaps a gain instead of a loss in clearness. The reason for this unusual brevity is not so much the compactness of the vector notation as the concreteness of the vector idea. No time is wasted in developing analytical formulæ. The mind is brought to bear directly upon the geometric questions at hand, and they are solved. It is this constant appeal to visualization that shortens the work; it is this that distinguishes the book from others; it is this that adds so much of perspicuity; and it is this that leads us to recommend the book most heartily to all who teach or study this subject. The vector ideas will clarify and render definite the conceptions of geometry to an extent scarcely possible with other methods. The insignificant amount of vector analysis need cause no fear of difficulty.

Mr. Fehr is to be thanked for his excellent book, and Messrs. Carré et Naud for the clear and almost errorless

typography. The following reproduction of the table of contents will serve to show more satisfactorily than anything else the scope of the work : *Préface, Introduction* : Rappel de quelques notions de calcul géométrique ; *Des courbes gauches* : 1° Généralités, 2° Courbure et rayon de courbure, 3° Torsion, formules de Frenet, 4° Courbure normale, formule de Lancret ; *De la théorie des surfaces* : 1° Généralités, 2° Relations fondamentales ; *De la courbure des courbes tracées sur une surface* : 1° Théorème de Meusnier, 2° Courbure des sections principales, 3° Formule d' Euler ; *De la courbure des surfaces* : 1° Courbure totale, application aux surfaces réglées, 2° Courbure moyenne ; cas particulier, 3° Courbure moyenne quadratique ; *Des lignes tracées sur une surface* ; 1° Systèmes conjugués, 2° Lignes de courbure ; théorème de Dupin, 3° Lignes asymptotiques, 4° Lignes géodésiques ; courbure géodésique d'une ligne tracée sur une surface.

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Annuaire pour l'Ar 1901, publié par le Bureau des Longitudes.
Paris, Gauthier-Villars.

THE new *Annuaire* which has just been received contains this time but few alterations. In the astronomical portion, however, the article on the various kinds of calendars has been rewritten and enlarged, the rules for obtaining the date of Easter in any year being much improved in statement and completeness. The historical part containing the tables of small planets, comets, etc., is, of course, brought up to date. A useful addition is a curve showing graphically the amount of the equation of time for any period of the year ; this has been carefully drawn by M. Cornu. In the scientific notices, M. Cornu resumes his article of last year under the title 'Le transport électrique de la force' ; M. Poincaré gives the report of the committee appointed to consider the question of the revision of the Quito meridian arc ; M. Bassot writes the history of the introduction and establishment of the metric system. MM. Loewy and Bouquet de la Grye respectively contribute accounts of the international astronomical and geodetic congresses held in Paris during the summer ; the aeronautical congress is represented by M. J. Janssen's presidential address. The last named also gives his annual report of the work done in the Mont Blanc Observatory during the past year. The final notice contains the oration of M. le commandant Guyou delivered at the meeting held to commemorate the late M. de Bernardières.