THE SEVENTH ANNUAL MEETING OF THE AMERICAN MATHEMATICAL SOCIETY.

THE Seventh Annual Meeting of the AMERICAN MATHEMATICAL SOCIETY was held in New York City on Friday, December 28, 1900. An important feature of the occasion was the election of officers and other members of the Council, at which recently adopted amendments of the Constitution went into effect. The term of office of the President, who must have served previously, but with a year's interval, as Vice-President, is now increased to two years. Both Presidents and Vice-Presidents are made ineligible for immediate reëlection. Four members of the Council, instead of three, are now elected annually. This provision and the permanent membership of the ex-Presidents increases the number of seats in the Council to twenty-four at present. The growth of the Society has been such that the numerical basis of representation of the Council is almost precisely the same as that of six years ago, when the Society was reorganized as a national body.

At the close of the century a brief recapitulation of the advance of the Society during these six years is not inappropriate. The record reveals in the past a happy and substantial augury for the future. The membership has grown from 244 in September, 1894, to 357 on January 1, 1901. In 1894 the number of papers read at the meetings was 24; in 1900 it was 112. This expansion in numbers and output has had for one valuable result the creation, in 1898, of the Chicago Section of the Society, an event which has proved not only a great benefit to many members to whom the New York meetings are practically inaccessible, but a source of strength to the Society as a whole. The demand for improved facilities for publication, occasioned by the great increase in productive activity of the members, has had for its outcome the founding of the *Transactions*, which has now successfully completed its first year of existence as the official organ of the Society for the publication of important original papers read before it. Meanwhile the BULLETIN has been considerably increased in size, although confining itself more strictly than before to the historical and critical field for which it was originally designed. A gratifying recognition of the usefulness and efficiency of the Society is evidenced by the liberal financial coöperation of ten leading universities of the country in the publication of the

Transactions. Since 1894 summer meetings, of great interest and largely attended, have been held, usually in connection with large and general scientific gatherings. On two occasions the summer meeting has been supplemented by colloquia, or special courses of lectures summarizing important fields of recent investigation. The attendance at the meetings throughout the year has constantly increased. In 1900 the total attendance was 209, and the number of members attending at least one meeting during the year was 110. The administration of business has been wisely left from the beginning in the hands of the Council, the time of the meetings being thus economized for purely scientific purposes. The finances of the Society are in a satisfactory condition, and the extra expense of publishing the *Transactions* has been met without a reduction of the small surplus which has slowly accumulated in the course of years. Over one-third of this surplus consists, however, of life membership payments, which have been set apart as a separate invested fund to be devoted to such purposes as may hereafter be approved. The Society is accumulating from the exchanges of the BULLETIN and the *Transactions* and from numerous gifts a considerable library of mathematical journals and works. It is to be hoped that this collection may be bound and made accessible to the members generally within a short time. The Society, however, has at present no funds available for this purpose.

The attendance at the annual meeting numbered about forty persons, including the following thirty-three members of the Society :—

Dr. C. L. Bouton, Professor E. W. Brown, Professor F. N. Cole, Dr. W. S. Dennett, Professor T. S. Fiske, Mr. W. B. Ford, Mr. A. S. Gale, Professor Harris Hancock, Dr. G. W. Hill, Dr. A. A. Himowich, Dr. J. I. Hutchinson, Mr. S. A. Joffe, Dr. Edward Kasner, Mr. C. J. Keyser, Professor Pomeroy Ladue, Dr. Emory McClintock, Dr. James Maclay, Professor Frank Morley, Dr. D. A. Murray, Professor W. F. Osgood, Mr. J. C. Pfister, Professor M. I. Pupin, Professor J. K. Rees, Professor C. A. Scott, Professor P. F. Smith, Dr. Virgil Snyder, Dr. W. M. Strong, Professor E. B. Van Vleck, Professor J. M. Van Vleck, Professor A. G. Webster, Professor L. A. Wait, Mr. E. B. Wilson, Professor F. S. Woods.

The meeting extended as usual through a morning and an afternoon session. Vice-President Thomas S. Fiske occupied the chair. The Council announced the election of the following persons to membership in the Society :—Dr. G. N.

Bauer, University of Minnesota, Minneapolis, Minn.; Dr. J. R. Benton, Princeton University, Princeton, N. J.; Dr. H. F. Blichfeldt, Leland Stanford University, Palo Alto, Cal.; Dr. G. A. Bliss, University of Minnesota, Minneapolis, Minn.; Professor Joseph Bowden, Adelphi College, Brooklyn, N. Y.; Professor D. F. Campbell, Armour Institute, Chicago, Ill.; Dr. J. E. Clarke, Gilbert School, Winsted, Conn.; Dr. Euplio Conoscente, New York, N. Y.; Mr. Arthur Crathorne, University of Wisconsin, Madison, Wis.; Mr. H. W. Curjel, Birkdale, Lancashire, Eng.; Professor L. M. Defoe, University of Missouri, Columbia, Mo.; Mr. B. S. Easton, University of Pennsylvania, Philadelphia, Pa.; Dr. L. P. Eisenhart, Princeton University, Princeton, N. J.; Mr. U. S. Hanna, University of Indiana, Bloomington, Ind.; Mr. L. I. Hewes, Yale University, New Haven, Conn.; Mr. A. M. Kenyon, Purdue University, Lafayette, Ind.; Professor C. N. Little, Leland Stanford University, Palo Alto, Cal.; Mr. E. L. Milne, University of Illinois, Champaign, Ill.; Mr. H. B. Mitchell, Columbia University, New York, N. Y.; Dr. Asutosh Mukhopâdyây, Calcutta, India; Mr. W. S. Nichols, New York, N. Y.; Professor J. M. Page, University of Virginia, Charlottesville, Va.; Dr. T. H. Taliaferro, Pennsylvania State College, State College, Pa.; Miss R. H. Vivian, University of Pennsylvania, Philadelphia, Pa., Mr. N. R. Wilson, Royal Military Academy, Kingston, Canada. Six applications for membership in the Society were reported. Reports were received from the Secretary, Treasurer, Librarian, and the Auditing Committee. These reports have been printed in the Annual Register, just issued.

At the annual election the following officers and members of the Council were chosen:

President,	Professor ELIAKIM H. MOORE.
Vice-Presidents,	Professor T. S. FISKE, Professor H. S. WHITE.
Secretary,	Professor F. N. COLE.
Treasurer,	Dr. W. S. DENNETT.
Librarian,	Professor POMEROY LADUE.

Committee of Publication,

Professor F. N. COLE,
Professor ALEXANDER ZIWET,
Professor FRANK MORLEY.

Members of the Council to serve until December, 1903,

Professor E. W. BROWN,
Professor H. B. FINE,
Professor T. F. HOLGATE,
Professor W. F. OSGOOD.

Member of the Council to serve until December, 1902,

Professor E. W. HYDE.

The following papers were presented at this meeting:

(1) Dr. VIRGIL SNYDER: "On some plane curves having factorable parallels."

(2) Professor E. D. ROE: "On a formula of interpolation."

(3) Mr. W. B. FORD: "Dini's method of showing the convergence of Fourier's series and of other allied developments."

(4) Dr. EMORY MCCLINTOCK: "A simplified solution of the cubic."

(5) Professor W. F. OSGOOD: "On the existence of a minimum of the integral

$$\int_{x_0}^{x_1} F(x, y, y')dx$$

when x_0 and x_1 are conjugate points, and the geodetics on an ellipsoid of revolution;" a revision of a theorem of Kneser.

(6) Mr. C. J. KEYSER: "Theorems concerning positive definitions of finite assemblage and infinite assemblage."

(7) Professor M. I. PUPIN: "Wave propagation over bridged wave conductors."

(8) Professor F. MORLEY: "On a point in Sylvester's theory of canonical forms."

(9) Professor HARRIS HANCOCK: "On primary prime functions in several variables and a generalization of an important theorem of Dedekind."

(10) Dr. J. I. HUTCHINSON: "On some birational transformations of a Kummer surface into itself."

(11) Miss R. G. WOOD: "The collineations of space which transform a non degenerate quadric surface into itself."

(12) Professor H. E. SLAUGHT: "The complete form system of invariants of the group of 120 quadratic Cremona transformations of the plane."

(13) Dr. JAMES MACLAY: "Some geometrical theorems conected with a class of differential equations derived from Poisson's equation

$$\frac{\partial^2 z}{\partial t^2} = a^2 \left(\frac{\partial^2 z}{\partial x^2} - \frac{mz}{x^2} \right).\text{''}$$

(14) Mr. J. K. WHITTEMORE: "Cones of the second degree osculating developable surfaces."

(15) Professor E. O. LOVETT: "The types of line-sphere transformation."

(16) Professor E. O. LOVETT: "Note on differential geometry of n-dimensional space."

(17) Dr. L. P. EISENHART: "A demonstration of the impossibility of a triply asymptotic system of surfaces."

(18) Professor MAXIME BÔCHER: "Some cases in which the identical vanishing of the Wronskian is a sufficient condition for linear dependence."

In the absence of the authors, Mr. Whittemore's paper was read by Professor Osgood, and the papers of Professor Roe, Professor Slaught, Professor Lovett, Dr. Eisenhart, and Professor Bôcher were read by title. Dr. Eisenhart's paper appeared in the January number of the BULLETIN. The papers of Mr. Ford, Mr. Keyser, and Dr. Hutchinson are included in the present number. Abstracts of the other papers are given below.

Dr. Snyder's paper is summarized as follows: Taking a scroll contained in a linear congruence and of the type having factorable asymptotic lines, one can transform the directrices into two anti-parallel planes in sphere space, and the generators into spheres which envelope a tubular surface. The lines of curvature are cut from this surface by a series of parallel planes. These sections are projections of parallels to the locus of centers of the surface, and break up into two factors. The same problem is then solved, starting from a surface of revolution, without using line geometry.

Professor Roe extends the formula

$$f(x + y) = (1 + \Delta)^y f(x),$$

for a positive integral y, to all real values of y within the limits of convergency and for the simplest expansion of the symbol $(1 + \Delta)^y$, by the immediate and direct relation of two series of the same function to each other, and independently of any Taylor series. Both symbolic and non-symbolic proofs are given. The following symbolic proof is given for the extension to a positive fractional commensurable y. Two series of the same function are taken

$$f(x+0)=u_0,\ f(x+1)=u_1,\ \cdots,\ f(x+r)=u_r,\ \cdots$$

with differences Δu's,

$$f(x+0)=v_0,\ f\left(x+\frac{1}{\lambda}\right)=v_1,\ \cdots,\ f\left(x+\frac{\nu}{\lambda}\right)=v_\nu\ \cdots$$

with differences δv's.

ν and λ are integers. It is shown that $v_\nu=(1+\delta)^\nu v_0$; then it is easily found that $\Delta^r u_0=[(1+\delta)^\lambda-1]^r v_0$; hence, with the confined meaning of $(1+\Delta)^{\frac{\nu}{\lambda}}$, that

$$(1+\Delta)^{\nu'}u_0=u_0+\binom{\nu'}{1}\Delta u_0+\binom{\nu'}{2}\Delta^2 u_0+\cdots+\binom{\nu'}{r}\Delta^r u_0+\cdots$$

$$=v_0+\binom{\nu'}{1}[(1+\delta)^\lambda-1]v_0+\binom{\nu'}{2}[(1+\delta)^\lambda-1]^2 v_0$$

$$+\cdots+\binom{\nu'}{r}[(1+\delta)^\lambda-1]^r v_0+\cdots$$

$$(\text{where } \nu'=\nu/\lambda)$$

$$=[1+(1+\delta)^\lambda-1]^{\nu'}v_0=(1+\delta)^{\nu'\lambda}v_0=(1+\delta)^\nu v_0$$

$$=v_\nu=f\left(x+\frac{\nu}{\lambda}\right),\ \text{ or }\ f\left(x+\frac{\nu}{\lambda}\right)=(1+\Delta)^{\frac{\nu}{\lambda}}f(x).$$

Extension is then made to an integral negative y, next to a negative fractional commensurable y, and lastly by limits to an incommensurable value of y, thus securing the extension to all real values of y for a convergent expansion.

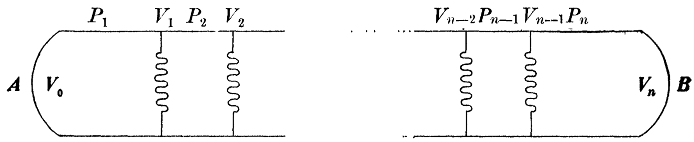
Dr. McClintock's "simplified solution of the cubic" is essentially a rearrangement, in a strikingly simple form, of Bézout's "earlier solution," with certain novel features. It will appear in the *Annals of Mathematics.*

In a paper entitled "Zur Variationsrechnung" (*Mathematische Annalen*, volume 50 (1897), p. 50), Kneser enunciates the theorem that the integral

$$I=\int_{x_0}^{x_1}F(x,y,y')dx$$

ceases to be a minimum, not only when the interval (x_0, x_1) contains in its interior a point x' conjugate to x_0, but when x_1 coincides with x'. This theorem is true in general, but not in all cases, and it is the object of Professor Osgood's paper to point out a general class of cases for which it is not true. The geodetics on an ellipsoid of revolution are a case in point. The paper will appear in the *Transactions*.

In Professor Pupin's paper the wave conductor is the loop AB in the figure. The bridges are denoted by 1, 2, $\cdots$, $n-2$, $n-1$. They divide the loop into n equal parts. Let l be



the distance between the bridges; L_1, R_1 the inductance and resistance, respectively, of each bridge; L, R, C, the inductance, resistance, and capacity, respectively, per mile, of the wave conductor. Let P be the potential and y the current at any point. P is continuous and y has as many discontinuities as there are bridges. In any section like m we shall denote by P_m and y_m the potential and current at any point of the section. Denoting the distance of any point of this section from the beginning of the section by s, we shall have

$$P_m = V_m, \qquad y_m = x_{2m+1}, \qquad \text{when } s = l;$$

$$P_m = V_{m-1}, \qquad y_m = x_{2m}, \qquad \text{when } s = 0.$$

The problem can now be stated as follows: Find the integral of the differential equation

$$L\frac{d^2P}{dt^2} + R\frac{dP}{dt} = \frac{1}{C}\frac{\partial^2 P}{\partial s^2}$$

which will satisfy n boundary conditions of the type

$$h(x_{2m-1} - x_{2m}) - 2V_m = 0,$$

where $h = ipL_1 + R_1$, and p is the frequency speed of the impressed simple harmonic electromotive force. The solution is

$$V_m = \frac{E}{2}\,\frac{\sin 2(m-n)\theta}{\sin 2n\theta},$$

where

$$\cos 2\theta = \cos \mu l - \frac{\mu \sin \mu l}{ipCh}, \qquad \mu^2 = p(pL - iR),$$

$$P_{m+1} = \frac{V_m \sin \mu(l-s) + V_{m+1} \sin \mu s}{\sin \mu l},$$

$$y_{m+1} = \frac{2ipC}{\mu \sin \mu l}\{ V_{m+1} \cos \mu s - V_m \cos \mu(l-s)\}.$$

When the bridges are sufficiently near with respect to the wave length which corresponds to the frequency speed of the force on the wave conductor without the bridges, then the system acts like a uniform wave conductor the capacity of which depends on the frequency. There is a critical frequency at which the wave length becomes infinite.

Professor Morley's paper is in abstract as follows: In the *Philosophical Magazine* for 1851 Sylvester gave a canonical form for a binary quantic of even order, as far as the eighth order, and suggested a general form which involved the finding of a certain covariant V. It seems to have been supposed that the difficulty of continuing lay in finding V (see Elliott, Algebra of quantics, p. 299). An examination shows that there is no theoretic difficulty, but that the source of V is easily obtainable as a determinant. The real objection to the theory lies in the complication of the expression of V by means of the fundamental covariants. Thus for the quantic of the tenth order, in terms of the covariants of Sylvester's auxiliary quintic, we have, in Elliott's notation for the quintic,

$$V = 100 C_{64} C_{51} - 25 C_{80} C_{35} + 3 C_{40}^2 C_{35} + 120 C_{71} C_{22}^2 + 92 C_{40} C_{75} + 24 C_{40} C_{22} C_{53}.$$

This formula was calculated by Mr. Coble.

Dedekind (in Supplement XI to Dirichlet's Zahlentheorie, 4th edition, p. 570) shows, if ω is any algebraic integer in a fixed realm of rationality Ω, that

$$\omega^{p^f} \equiv \omega \pmod{\mathfrak{p}},$$

where p is a rational prime integer that is divisible (in Ω) by the prime ideal $\mathfrak{p}$ and f is the degree of $\mathfrak{p}$. He then shows, pp. 570–572, that

$$t^{p^f} - t \equiv \Pi(t-\omega) \equiv \Pi P(t) \pmod{\mathfrak{p}},$$

where t is a variable and where the first product is taken over a system of incongruent (mod. $\mathfrak{p}$) algebraic integers in Ω and the second product is taken over all primary prime functions $P(x)$, whose degree is f or a divisor of f. The investigations are restricted to algebraic numbers in a fixed realm. Professor Hancock considers instead of the algebraic numbers integral functions in any number of variables whose coefficients are the Dedekind algebraic numbers; instead of the prime ideal $\mathfrak{p}$ there occur in his general treatment prime modular systems, in whose elements enter all the variables save one. Primary prime functions are taken in which the variable that is wanting in the modular system occurs to degree h and where the coefficients are integral functions of the other variables that are found in the modular system. The number of such primary prime functions whose degree is h or a divisor of h is calculated, and then the generalized Dedekind's theorem is developed.

Miss Wood's paper deals with the collineations of space which transform a non-degenerate quadric into itself. Starting with the theorem that such a collineation which leaves the two systems of generators invariant may be compounded of two skew reflections, the following theorems in non-euclidean geometry are deduced. For elliptic space: 1° If a straight line be displaced, the loci of the middle points of chords joining congruent points are straight lines. 2° Planes perpendicular to these chords at their middle points intersect each set in a straight line. 3° If a plane figure be displaced, the loci of the middle points of chords joining congruent points are planes. 4° Planes perpendicular to these chords at their middle points pass each set through a point. For hyperbolic space: 1° If a straight line be displaced, the locus of the middle point of chords forming congruent points is a straight line. 2° Planes perpendicular to these chords at their middle points intersect in a line. 3° If a plane figure be displaced, the middle points of chords joining congruent points lie in a plane. 4° Planes perpendicular to these chords at their middle points pass through a point in the plane. When the quadric degenerates four well known theorems of Chasles for euclidean space are obtained.

In a paper soon to be published in the *American Journal of Mathematics*, Professor Slaught has discussed the nature of invariants of a quadratic group and has shown that, inasmuch as the degree of any rational integral form is doubled under a quadratic transformation, only a rational fraction can transform into itself after throwing off a common factor in the variables from numerator and denominator. For the quadratic Cremona group of 5! cross ratio transformations it is shown that the most general forms suitable for numerator and denominator of such invariant fractions are of degree $6n$ and throw off a factor $(z_1z_2z_3)^{2n}$ under the quadratic generator

$$z_1' : z_2' : z_3' = z_2z_3 : z_1z_3 : z_1z_2.$$

Such a form, R_{6n} is found to be decomposable into two factors

$$R_{6n} = P^{2\mu} R_{6(n-2\mu)}$$

in which

$$P = z_1z_2z_3(z_1 - z_2)(z_1 - z_3)(z_2 - z_3),$$

$\mu = 0$ or a positive integer, and $R_{6(n-2\mu)}$ contains *no factor* of P.

The factors of P, set equal to zero, are the six sides of the complete quadrangle whose vertices are the four critical points of the group, and every invariant curve

$$R_{6n} = 0$$

has at each of these critical points a multiple point of order $2(n + \mu)$.

The quadratic group G_{120} has a linear subgroup $G_{24}^{(1)}$ which is projectively related to the Klein's linear group of order 4!, and through a study of the invariants of these two linear groups there are found the most general invariant forms under G_{120} of degree 6, 12, and 18 [$n = 1, 2, 3$], namely,

$$mA, \quad m_1A^2 + m_2P^2, \quad m_1A^3 + m_2AP^2 + m_3C,$$

in which the m's are arbitrary and A, P^2, C are three invariants of the binary quintic form, written in terms of the cross ratios of the five roots of the quintic. Then by a series of theorems relating the general forms R_{6n} to the geometry of the group [The geometric representation was given in the October number of the *American Journal of Mathematics.*] it is shown, through a process of successive reduction, that any invariant form under G_{120} is expressible as a rational integral function of A, P^2, C, and thus a system of fundamental forms is established in terms of which all invariant fractions under the quadratic group can be expressed.

As is known, any three solutions of the differential equation

$$\frac{\partial^2\vartheta}{\partial u\partial v} + f(u, v)\frac{\partial\vartheta}{\partial u} + \varphi(u, v)\frac{\partial\vartheta}{\partial v} = 0,$$

when set equal to x, y, z, respectively, define a surface referred to a system of conjugate lines. Dr. Maclay determines a transformation of a character to change conjugate systems of curves on a surface into conjugate systems on the transformed surface. After some discussion of the relations of a surface and its transformed in the general case, the surfaces defined by

$$f = \varphi = \frac{n}{u + v}$$

and the corresponding transformed surfaces are more particularly studied. By suitable transformations the latter equation is reduced to the form given in the title of the paper, and the complete integral is obtained. The transformation, when employed upon the general surface of translation, will change the curves of translation into lines of curvature on the transformed surface only in the case of the right circular cylinder, and the resulting surface is any surface of revolution.

Mr. Whittemore's paper gives a short account of Hermite's investigations on quadric surfaces osculating a given surface. Hermite showed that, in general, at a finite number of simple points at any surface, the surface is osculated by all the members of a one parameter family of quadrics. In the present paper it is proved that a developable surface is osculated at every simple point by all the members of a one parameter family of quadrics. It is then shown that all the quadrics osculating a developable at any point are cones having a common vertex which is on the edge of regression of the developable. A complete geometrical definition of these cones is then obtained.

In a recent note Professor Lovett has shown that the various contact transformations of ordinary space which change straight lines into spheres are determined by pairs of bilinear equations between the point coördinates of the corresponding spaces, and that they fall into two categories according as all or certain determinants of a certain matrix reduce to constants. In the first of Professor Lovett's present papers the explicit forms of all the transformations of one category are given. There are thirty-one types, two of which depend on solutions of equations of the sixth degree; three on quintics; one on a pair of quartics; two on cubics; two are linear; and the remainder quadratic. To satisfy the equations of condition for the second cate-