Vorlesungen über Kreis- und Kugel-Functionen-Reihen. Von DR. JOHANNES FRISCHAUF. Leipzig, B. G. Teubner, 1897, pp. 60.

THIS little treatise will be welcomed by students of pure analysis and also by students of mathematical physics. The subject treated has already an extensive literature of its own; but the important results in this field are scattered through different memoirs of Dirichlet, Du Bois Reymond, Bonnet, Lipschitz, Dini, and others, while there has appeared no distinctly elementary treatment of the subject as a whole. Thorough and comprehensive treatments of Fourier's series and other allied developments have indeed been given by C. Neumann in his "Uber die nach Kreis- Kugel- und Cylinder-Functionen fortschreitenden Entwickelungen," and by Dini in his "Serie di Fourier e altre rappresentazioni analitiche delle funzioni di una variabile reale," but these treatments could hardly be called elementary—certainly not in comparison with the above work. The author has followed a uniform and easy method throughout and has shown the convergence of Fourier's series, of Fourier's integral, and of the developments in terms of zonal and spherical harmonics in case the function to be developed has but a limited number of maxima and minima within a certain interval and is either continuous within this interval or else has but a finite number of so called "finite discontinuities." Some of the simpler cases are also considered in which the maxima and minima become infinite in number in the neighborhood of certain points, or in which the function at certain points becomes infinite in value. In brief, the author has given us an elementary, systematic treatment of these series for all cases which might ordinarily arise in connection with nature.

Chapter I. deals especially with Fourier's series. Chapters II. and III. are devoted to the discussion of certain general properties of zonal and spherical harmonics, and Chapter IV. relates to the developments in terms of the latter functions. Illustrative examples are given throughout.

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