Then (I) shows that Ω is a solution of the partial differential equation

$$X\frac{\partial f}{\partial x} + Y\frac{\partial f}{\partial y} + Z\frac{\partial f}{\partial z} = 0,$$

equivalent to the simultaneous system (1).

Evident misprints occur on p. 145, l. 7, p. 157, p. 182. It adds clearness to use y cot nx instead of cot nxy used p. 188.

A final remark is that it seems preferable to teach a general *method of procedure* for solving differential equations using freely transformations of the independent and dependent variables, rather that the application of a general formula. For example, the integration of the general linear differential equation of the first order is performed by a simple method, but by a complicated formula.

L. E. DICKSON.

UNIVERSITY OF CALIFORNIA,
March 23, 1899.

TANNERY'S ARITHMETIC.

Leçons d'Arithmétique théorique et pratique. By JULES TANNERY. Paris, Colin et Cie, 1894. viii+509 pp.

THE present volume from the pen of the distinguished director of scientific studies at the École Normale Supérieure in Paris is the first work on arithmetic we have seen which while intended entirely for secondary instruction is written in accordance with the new ideas regarding the number concept and the need of rigor. It is thus a pioneer, perhaps even the inaugurator, of a revolution in secondary instruction in mathematics and as such will receive praise or censure according as the person in question is thoroughly awake to the crying necessity of reform in secondary mathematical instruction, or is not.

For fifty years or more slow changes have been taking place in the mathematical world. Their cumulative effect has completely transformed the aspect of mathematics from its bottommost foundations to the summit. Such mathematicians as Gauss, Cauchy, and Abel found the great structure of mathematics almost without foundation. Here is an extract of a letter of Abel to Hansteen, dated 1826: "Je consacrerai toutes mes forces à répandre de la lumière sur l'immense obscurité qui règne aujourd'hui dans l'analyse. Elle est tellement dépourvue de tout plan et de tout système qu'on s'étonne seulement qu'il y ait tant de gens qui s'y livrent—et ce qui est pis, elle manque absolument de rigueur." In a letter to Holmboe he writes: "Enfin mes yeux sont dessillés d'une maniére frappante, car à l'exception des cas les plus simples, par example les séries géométriques, il ne se trouve dans les mathématiques presque aucune série infinie dont la somme soit determinée d'une manière rigoureuse, c'est à dire que la partie la plus essentielle des mathématiques est sans fondement." Under the leadership of Gauss, Cauchy, and Abel a great reform set in. The most important factor in this process was the number concept. In order to gain a basis on which the magnificent theories of our predecessors could be securely based it was found in last analysis that the difficulties were purely arithmetical. To overcome these difficulties it was necessary to create a new theory of number, quantity, and magnitude. Today we have not only precise ideas on continuous number but also a satisfactory theory of extensive number or *polyplets* of which complex numbers are the simplest type, as well as a theory of magnitude in general. These great achievements have as yet had no influence on text books of secondary grade. The arithmetics, algebras, trigonometries, and works on calculus, with a few conspicuous exceptions, are no whit better in this respect than they were a generation ago. The absolutely false, or lamentably imperfect and inadequate ideas of the analysts of the last century we find repeated *ad nauseam* in nearly every new text book on these subjects.

In our opinion the time has come to sound an alarm, to awaken our educators to the fact that it is not just or wise to teach our youths notions which, if not altogether obsolete, have been so profoundly transformed as to be hardly recognizable. It is not the circle of ideas of the last century that we should instill in their minds but those of this end of the nineteenth century. What we wish to see is a series of text books which begins at the beginning, viz., arithmetic, conducts our youth through our college mathematics, and which is written with due attention to the present state of mathematics and the present demands made in regard to rigor.

To have advocated such a course before the appearance of Tannery's *Leçons* would no doubt have seemed ridiculous to many. We should have been overwhelmed by *à priori* arguments drawn from the science of pedagogics against its possibility. Venerable colleagues, grown white in the service of instruction, would have given us up as dreamers of impracticable things. To all these and to those who are truly eager for progress, who wish to see secondary mathematical instruction in this country abreast of the times we say: read the *Leçons* and then reflect.

The most difficult part of our program is the first volume in the series, the arithmetic. How to present our modern notions on this subject clearly and rigorously has seemed to us, who have long cherished these ideas, a task so arduous that few could hope to attain even fair success. And yet a correctly yet simply written arithmetic is the key to the situation, a *sine qua non*. We consider it a fortunate circumstance for the cause of a better secondary instruction in mathematics that the first attempt in this direction fell to no less able hands than those of M. Tannery. We confess gladly that his *Leçons* have been a revelation to us. He has solved so many of the difficulties which beset his path with such admirable tact that we believe future authors will be obliged to follow largely the path he has opened. At any rate the *Leçons d'Arithmêtique* are for us an ocular demonstration that a series of text books such as we have contemplated is not only possible but desirable. We believe that other mathematicians will now be encouraged by M. Tannery's success and that the time is not far distant when our mathematical text books of secondary grade will be as excellent and as up to date as those written today in connection with university instruction.

We extend our sincere congratulations to M. Tannery for his bold and successful innovation. If we have not reviewed his book in detail it is because we have believed that we could employ the limited space at our disposal to best advantage in signalizing its nature and its real importance.

JAMES PIERPONT.

YALE UNIVERSITY,
May, 1899.