NOTE ON SPECIAL REGULAR RETICULATIONS.

BY PROFESSOR ELLERY W. DAVIS.

As in Professor H. S. White's first paper (BULLETIN, December, 1896) I write

$$\rho = r - 2, \quad \sigma = s - 2, \quad F = (4p - 4)(\rho + 2) / (\rho\sigma - 4).$$

With the exception of cases like 5_{14}, 14_5 and 6_{11}, 11_6 for $p = 9$, it appears from the tables and is a direct implication of Professor White's mode of building up the reticulations, that whenever for a given p we have an F less than its corresponding V, then there will also be found the same F paired with a V at most equal to it. But since $Fs = Vr$, this is the same as saying that when $\rho < \sigma$ for a given F, for the same F we can also have $\rho \geqq \sigma$.

Let $\sigma = \rho + K$. Then

$$K = \frac{1}{F\rho}\,[4(2p - 2 + F) + (4p - 4)\rho - F\rho^2]$$

and ρ must divide $4(2p - 2 + F)$. If $2p - 2 + F = \delta_1\delta_2$, $\rho = \delta_1$ or $2\delta_1$ or $4\delta_1$ where δ_1 is any factor of $2p - 2 + F$ including both unity and $2p - 2 + F$.

$$\rho = \delta_1 \text{ gives } K = K_1 = \frac{1}{F}\,(\delta_1 + 2)(2\delta_2 - F), \quad V = 2\delta_2.$$

$$\rho = 2\delta_1 \text{ gives } K = K_2 = \frac{2}{F}\,(\delta_1 + 1)(\delta_2 - F), \quad V = \delta_2.$$

$$\rho = 4\delta_1 \text{ gives } K = K_3\,\frac{1}{F}\,(2\delta_1 + 1)(\delta_2 - 2F), \quad V = \delta_2/2.$$

When F, and therefore δ_2, is odd, K_3 makes V a fraction; otherwise we have our choice of the three K's. Should, for a given F and p, one of the K's become a positive integer but none a negative integer, no matter what factor were taken of $2p - 2 + F$, an exceptional reticulation would be before us.

Plainly when $F = 1$ or 2 we can always have K either zero or negative. When $F = 2$, however, and p is odd, F and $p - 1$ have the common factor 2 so that the reticulation is a derived one unless V is odd. If we put $p = 2s + 1$, $\delta_2 = 1$, we find $K_2 = -4S - 1$, and $V = 1$ so that the reticulation is special. Thus for $F = 1$ or 2 it can never happen that there shall be a positive K without a negative one.

Similarly, if $F = 3$ there is no special reticulation for $p = 3s + 1$ while for $p = 3s$ or $3s - 1$ there is always one with a negative K.

When $F = 4$ we can have $K_1 = -p - 3$ belonging to a special reticulation when p is even, while for p odd there is no special reticulation.

When $F = 5$ we finally get exceptional reticulations provided $p = 5s - 1$ or $5s + 2$ and the s is rightly chosen. The simplest is that in Professor White's table, 5_{14}, 14_5.

Again, when $F = 6$ there are sometimes exceptional reticulations for $p = 6s + 3$. The simplest is again one given in Professor White's tables 6_{11}, 11_6.

Other special reticulations occur for $F = 7$. The simplest is 7_{10}, 10_7 for $p = 10$.

In all attempts to realize these exceptional reticulations by construction I have failed. Nor do I see any way of proving that they cannot be constructed. This last once done would show Professor White's method to be exhaustive.

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LIMITATIONS OF GREEK ARITHMETIC.

BY MR. H. E. HAWKES.

(Read before the American Mathematical Society at the Meeting of April 30, 1898.)

I PROPOSE to discuss in the present paper the number system of the Greeks, and to show how their arithmetical notions were limited by their geometrical symbolism. My argument is based chiefly on Euclid's Elements. This is not a serious limitation, for, firstly, the Elements give us practically all that Greek mathematicians knew on the subject, prior to 300 B. C., and, secondly, little was accomplished in this direction during the following three or four centuries. We may, therefore, consider Euclid's theory of number as representative.

I shall first attempt to show that Euclid naturally ex-