

CHAPTER XI: Normal curves, Moduli. The chapter opens with the theorem: "There exists no birational transformation, containing an arbitrary parameter, that transforms a curve of deficiency greater than unity into itself." The most general transformation of a curve of deficiency unity into itself is obtained in terms of the parameter w . Clebsch's and Noether's normal curves are studied and the number of moduli of the algebraic configuration is determined by counting off parameters. Thus a lower limit to the number of the moduli is rigorously established and an elegant application of this result is made to the proof of the theorem that an algebraic plane curve with non-specialized coefficients is not the most general curve of its deficiency.

CHAPTER XII presents some geometric applications of Abel's theorem, principally those to the groups of points of intersection of curves. The points of inflection of a cubic and the points where a conic has contact of the fifth order, the systems of conics tangent to the quartic wherever they meet it, the double-tangents of the quartic, and the discussion of some of these topics when the ground-curve has double-points, are among the subjects treated; but a few examples in areas, angles and lengths of arcs are taken up at the end.

The reviewer has expressed his mind freely on those points where his views differ from those of the authors. He begs leave to say in closing that his admiration for their work is none the less hearty because of these differences of opinion and to add that their treatise seems to him to be a thoroughly desirable book to put into the hands of the student.

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HARVARD UNIVERSITY,
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PAPERS OF THE MATHEMATICAL CONGRESS.

Mathematical Papers Read at the International Mathematical Congress held in Connection with the World's Columbian Exposition, Chicago, 1893. Edited by the Committee of the Congress: E. HASTINGS MOORE, OSKAR BOLZA, HEINRICH MASCHKE, HENRY S. WHITE. New York, The Macmillan Company, for the American Mathematical Society. 1896. 8vo. xvi. and 411 pp. \$4.00.

THIS volume, which enjoys the distinction of being the first to be published in the form of a separate book by the

American Mathematical Society, has a number of claims to the attention of mathematical readers besides the intrinsic merits of the papers which it contains. One of these is the significance of the occasion which it commemorates. International Congresses bid fair to play an important rôle in the mathematical life of the future and the Chicago Congress is the initial gathering of the sort. Another is the highly international character of the collection itself, twenty-one of its papers having been contributed by German mathematicians, fourteen by American, four by French, three by Italian, two by Austrian and one by a Russian. And still another is the prominence in it of interesting *resumés* of recent progress in various departments of mathematical science, for the greater part admirably done by men who have themselves made important contributions to the departments of which they write. The success of the Congress itself was in large measure due to the presence and active participation of Professor Klein, of Göttingen. And the present volume owes to him not only the address made at the opening of the Congress on *The Present State of Mathematics*, but a number of papers by his university colleagues "which give collectively a fairly complete account of contemporaneous mathematical activity in Germany." Dr. Burkhardt, of Göttingen, discusses the recent developments in the *theory of perturbations*, examining the methods of Hill, Gylden and Linstedt from a mathematical point of view, particularly in their relation to the theory of the irregular integrals of linear differential equations. Dr. Fricke, now of Braunschweig, reviews the relations of the *theory of automorphic functions* to the higher arithmetic. Dr. Heffter, of Giessen, sketches the advances made in Germany in the theory of *linear differential equations* since the appearance of the epoch-making memoirs of Fuchs (in volumes 66 and 68 of *Crelle's Journal*). Professor Netto, of Giessen, expounds the thesis of *Kronecker* that it must ultimately be possible to "arithmetize" the whole of analysis, to reduce it to a body of relations expressible in finite terms among integral numbers, and his methods for accomplishing this reduction—the modular systems and the "association" with the algebraic functions of linear combinations of them with indeterminate coefficients.

Professor Schoenflies, of Göttingen, shows the significance of the theory of groups for modern *crystallography*. And Professor Study, now of Bonn, makes a review of "older and newer" investigation of systems of *complex numbers*, those especially in which these systems are considered in

connection with transformation groups. And besides these reviews by German mathematicians there is an account by Professor Eddy, now of the University of Minnesota, of the remarkable development of *graphical methods* within the past thirty years, particularly in connection with engineering theory and practice.

The same general interest attaches to the paper in which Professor Hilbert, now of Göttingen, gives an outline of his own important investigations in the field of *algebraic invariants*; to that of Professor Minkowski, now of Königsberg, on his *geometry of integral numbers*; of Professor Pringsheim, of Munich, on his general theory of the *divergency and convergency of series* with positive terms; of Professor Study on his researches in *spherical trigonometry*; and of Mr. D'Ocagne, of Paris, on his powerful graphical method: "*Nomographie*."

But the volume is not simply a collection of expository papers and *resumés*. It contains a full share of valuable and interesting original papers also. In the field of the *theory of substitutions* and *algebraic equations* are papers by Cole now of Columbia, Maschke and Moore of Chicago, de Perrott and Taber of Clarke and Weber of Strasburg; in the field of *functions*, papers by Bolza of Chicago, Hermite of Paris, Krause of Dresden, Pincherle of Bologna and Stringham of the University of California; while mathematical *history*, the reduction of *binary quantities*, *continuous transformation groups*, *algebraic curves*, the *Grassmann analysis* and *geodesic lines* are represented by papers by Halsted of the University of Texas, Hurwitz of Zürich, Meyer of Clausthal, Noether of Erlangen, Schlegel of Hagen and Weyr of Prague respectively. From every point of view, therefore, this volume is an important contribution to the mathematical literature of America. It is, moreover, carefully edited and beautifully printed.

HENRY B. FINE.

ELECTRICITY AND MAGNETISM.

Elements of the Mathematical Theory of Electricity and Magnetism.

By J. J. THOMSON, F. R. S., Cavendish Professor of Experimental Physics in the University of Cambridge. vi+510 pp., octavo. Cambridge, University Press; New York, The Macmillan Company, 1895. \$2.60.

When Professor Thomson undertook to edit a new edition of Clark Maxwell's treatise it was hoped he would re-