The length of the arc of an asymptotic curve is given by the integral

$$s = \sqrt{\lambda}\int_v^\lambda \frac{dv}{\sqrt{(\lambda - v)(1+\lambda - v)(1-\lambda+v)}}.$$

Introducing the $\wp$-function with the invariants $g_2 = \frac{1}{4\lambda^2}$, $g_3 = 0$, and $e_1 = \frac{1}{4\lambda}$, $e_2 = 0$, $e_3 = -\frac{1}{4\lambda}$, $\left(k^2 = \frac{e_2 - e_3}{e_1 - e_3} = \frac{1}{2}\right)$, we obtain:

$$\lambda - v = \frac{1}{4\lambda}\cdot\frac{1}{\wp s}.$$

UNIVERSITY OF CHICAGO, *August* 23, 1895.

ON A GENERALIZATION OF WEIERSTRASS'S EQUATION WITH THREE TERMS.

BY PROFESSOR F. MORLEY.

THE expression

$$\prod_{\lambda=1}^{n}\sigma(u - b_\lambda)/\sigma(u - a_\lambda)$$

is an elliptic function of u if

$$\Sigma a_\lambda = \Sigma b_\lambda.$$

The sum of the residues is zero; that is,

$$(1)\qquad \sum_{\lambda=1}^{n}\frac{\sigma(a_\lambda - b_1)\ldots\sigma(a_\lambda - b_\lambda)\ldots\sigma(a_\lambda - b_n)}{\sigma(a_\lambda - a_1)\ldots\quad 1\quad\ldots\sigma(a_\lambda - a_n)} = 0.$$

Being now only concerned with differences, we can, by a suitable addition to each a and b, write

$$(2)\qquad \Sigma a_\lambda = \Sigma b_\lambda = 0.$$

When $n = 2$, the equation (1) is in no way characteristic of the σ-function, but is true of any odd function.

When $n = 3$, (1) becomes

$$(3)\qquad \begin{aligned} &\sigma(a_1 - b_1)\sigma(a_1 - b_2)\sigma(a_1 - b_3)\sigma(a_2 - a_3)\\ +\ &\sigma(a_2 - b_1)\sigma(a_2 - b_2)\sigma(a_2 - b_3)\sigma(a_3 - a_1)\\ +\ &\sigma(a_3 - b_1)\sigma(a_3 - b_2)\sigma(a_3 - b_3)\sigma(a_1 - a_2) = 0.\end{aligned}$$

To identify this with the equation of three terms as given by Weierstrass, write

$$\begin{aligned} a &= b_1 - a_1, \\ b &= a_1 - b_2, \\ c &= a_1 - b_3, \\ d &= a_2 - a_3, \end{aligned}$$

and use the notation given by Harkness and Morley, Theory of Functions, p. 313.

We have

$$\begin{aligned} a' &= b_1 - a_3, & a'' &= b_1 - a_2, \\ b' &= a_3 - b_2, & b'' &= a_2 - b_2, \\ c' &= a_3 - b_3, & c'' &= a_2 - b_3, \\ d' &= a_1 - a_2, & d'' &= a_3 - a_1, \end{aligned}$$

so that our equation (3) may be written

$$\sigma a \sigma b \sigma c \sigma d + \sigma a' \sigma b' \sigma c' \sigma d' + \sigma a'' \sigma b'' \sigma c'' \sigma d'' = 0,$$

which is equation (48) of the page referred to.

For higher values of n the equation (1) appears to be the simplest of the various possible extensions of the equation with three terms.

NOTES.

Among recent academic appointments are the following: Professor Samuel M. Barton, recently of the Virginia Agricultural and Mechanical College, has been appointed professor of mathematics at the University of the South, Sewanee, Tenn.; Dr. Alexander Macfarlane, formerly professor of physics at the University of Texas, has accepted a lectureship in electrical engineering at Lehigh University, South Bethlehem, Pa.; Dr. E. B. Van Vleck has resigned an instructorship at the University of Wisconsin in order to become associate professor of mathematics at Wesleyan University, Middletown, Conn.; Professor C. A. Waldo has been called from De Pauw University to the chair of mathematics at Purdue University, La Fayette, Ind.; Professor W. J. Hussey, of Stanford University, has been appointed astronomer at the Lick Observatory, to succeed Professor E. E. Barnard, who has been called to a chair of astronomy at the University of Chicago.

Dr. Paul Stäckel, of Halle, has been appointed professor extraordinarius at Königsberg, and Professor V. Eberhardt, of Königsberg, has been called to Halle.

We are grieved to learn of the death of Dr. Ernst Ritter, on August 23, of typhoid fever, at the government hospital