In the chapter on Lamé's functions it is to be regretted that symmetry has not been preserved in relation to the three ellipsoidal co-ordinates, as has been most elegantly done in Halphen's "Traité des Fonctions Elliptiques."

The book is just what it purports to be. The preface states that the first part is based on Riemann-Hattendorff, and it includes besides a great deal not there treated. It is a clear, compact treatment of its subject-matter, and will be of great value to students of mathematical physics and to all persons who have to perform calculations of the kind considered. It contains those things that the "business" mathematical physicist wants to know, so arranged that he can find them at once. It is in addition much more interesting than such books have generally been. Heine's and Thomson and Tait's have been the standard treatises on spherical harmonics, but no one could pretend that Heine's was an attractive book to read, or Thomson and Tait's easy. Byerly's book is crowded with physical problems of all sorts, mostly worked out in detail. A good opportunity is also given the student to exercise himself in real numerical calculation by which he may get a tangible idea of the processes involved. A series of valuable tables of the values of the various functions is also given. Last, and not least in value, is to be mentioned the historical summary contributed by Dr. Maxime Bôcher, giving an admirable sketch of the whole subject, with a bibliography.

The book is well and clearly printed, and attractive in appearance (to one, as was stated at the beginning, who likes that sort of thing). Misprints are rare. On page 91 Ångström appears as Ängstrom, which spoils the pronunciation.

It may be mentioned that the historical essay on trigonometric series mentioned on page 61 is to be found in the *Bulletin des Sciences mathématiques* for 1880.

ARTHUR G. WEBSTER.

CLARK UNIVERSITY, WORCESTER, MASS.

NOTE ON SMITH'S REVIEW OF CAJORI.

BY PROF. GEORGE BRUCE HALSTED.

THE review, in the May BULLETIN, of Cajori's History of Mathematics by Professor David Eugene Smith produces an unfair impression. The facts upon which he says he bases his "harsh statement" do not justify it; and what he states as his "*facts*" are in large part not facts, but specimens of Professor Smith's *petitio principii*.

His *First* is, "The work is, in very considerable measure, merely a paraphrase of portions of *better* works." He then cites Ball and Fink, neither of which is a "better" work; in fact, apart from mere printer's slips in printing foreign words, Cajori is superior in point of actual accuracy to either of them, and incomparably superior in range.

Contrast Smith's *First* with his *Second:* "One has a right to expect a rich set of references to the standard literature of the day. Such references are offered by other histories, however humble, and every student needs them. Yet in this work there is not a single reference by volume and page." But neither is there in Fink, though just cited among those *better* than Cajori.

Such references were deliberately abjured by Cajori as interfering too seriously with the popular character and readableness of his book.

As for Smith's Dictionary of Biography, this is a slip of Professor Smith, see 9 under Books of Reference. Had Professor Cajori waited to obtain and study all the works of all the men whose names are so easy for Mr. Smith to mention, he might have died of old age, instead of giving us a charming history of mathematics.

That Mr. Smith himself is not above criticism I judge from the fact that he does not perceive the most serious error in Cajori's book, pointed out in the *Educational Review.*

Mr. Smith's *Third:* "But however charitable the reader may be, he will close the final chapters with even greater disappointment than he experienced in reading the earlier ones." I say, on the contrary, that these very chapters for successful condensed popular statement of the ground they cover are without rival in the world, in any language.

"While . . . thirty American mathematicians could not be found who would wish to be mentioned in a work which ignores the names of so many world-known promoters of the science;"—the word "ignores" for "omits" seems unfair, and 30 is perhaps a slip for 300.

His "*Fourth.* A final reason . . .: the work is carelessly written." This I beg leave to categorically deny. For example, Chasles' Christian name is given correctly. But it appears that "carelessly" is an exaggeration for "not with the *greatest* care."

"In the bibliography, . . . errors will be found in" . . . 65 [u for ü], 67, 84, 88, 97 [in all four the same small *t* omitted from the same word by the printer], 96 [B for R]. Thus the row of "numbers" is seen to be unfair.

In the words of the *Nation's* review: "But these are trifling faults. What we have a right to expect in such a handbook is an agreeable narrative of the most material events in the history of mathematics, and this Professor Cajori incontestably supplies."

When two reviewers hold such opposite opinions, and express their views in a manner so radically different, as Professor Halsted and the writer do concerning Cajori's History of Mathematics, the question becomes merely a matter for the reader to decide. It is submitted that Dr. Halsted's statements invalidate in no single point the review published in the May number.

As regards the first objection, the works of Ball and Fink are not the only, nor even the chief, ones referred to, as Dr. Halsted's article seems to imply. Indeed the quotation from Fink is rather to illustrate the faulty translation, that author not being mentioned among those from whom Professor Cajori has chiefly drawn. The works of Gow, Hankel, and Cantor, which will probably be admitted to be "better" works, are the ones to which, with Ball, the most prominent reference is made. But the real point of the criticism is not as to which of any two works is the better, but as to the way in which various works have been laid under contribution. If Professor Cajori's book contains such close paraphrases of inferior histories, so much the worse. While it is foreign to the real issue, the writer would not have it inferred from any silence of his that he for a moment subscribes to the proposition that "Cajori is superior in point of actual accuracy" to the second edition of Ball.

As to the second point, it is not clear why it should have interfered with "the popular character and readableness of his book" if such a bibliographical reference as that mentioned on p. 193 of the BULLETIN had been made more explicit, or if references had been given whenever an extract was made. Can any one justly affirm that the references as they stand are of any material value? Replying further to Professor Halsted, it is reasserted that the valuable *articles* (as distinguished from the valuable *article* mentioned) in Smith's Dictionary of Biography are worthy of mention. That the author should have "waited to obtain and study all the works of all the men" mentioned was not asserted. The criticism distinctly refers to the "sweeping omission"—to the fact that not one of them was consulted. But surely Professor Halsted, who has himself done so much for the history of mathematics in this country, will not contend but that every advanced scholar in this line is familiar with substantially every author named in the general list; and the professor must admit that one like himself could easily, with no hesitation, add several well-known names, and many of lesser note, to those mentioned.

The error pointed out in the article in the *Educational Review* can hardly, save on the principle of *ipse dixit*, be called "the most serious error in Cajori's book." It is, however, quite pardonable for Professor Halsted to be partial to that article; he wrote it. As to the article in the *Nation*, that was a charming essay, but it was hardly a serious review of Cajori.

Since with the exception of a few pages in Ball we have no similar attempt at a synopsis of the whole of modern mathematical history up to the present time, it is quite safe to make the sweeping assertion that this portion of the work is "without a rival in the world, in any language." Nevertheless one may close the final chapters with disappointment.

Having no idea of the meaning of Professor Halsted's reference to Chasles' Christian name, or of his statement that "30 is perhaps a slip for 300," the writer ventures to pass them by. He also ventures to reassert his appreciation of the work under discussion as a popular exposition of the historical advance of mathematical science, as set forth in the closing paragraph of his review.

DAVID EUGENE SMITH.

Michigan State Normal School,
YPSILANTI, MICH.

ON ORTHOGONAL SUBSTITUTIONS.

BY PROF. HENRY TABER.

IN 1846, in *Crelle's Journal*, Cayley gave his well-known determination of the general proper orthogonal substitution of n variables rationally in terms of the minimum number of parameters. Subsequently, in *Crelle*, vol. 50, and in the *Philosophical Transactions* for 1858, Cayley expressed these results in the notation of matrices.

In accordance with the theory of matrices,* two linear substitutions are regarded as susceptible of being added or subtracted. If $(\phi)_{rs}$ denotes that coefficient of the linear substitution ϕ which appears in the rth row and sth column of its square array or matrix, the sum or difference of the linear substitutions ϕ and ψ is defined as follows:

$$(\phi \pm \psi)_{rs} = (\phi)_{rs} \pm (\psi)_{rs}.$$

Addition and subtraction of linear substitutions are then subject to the laws which hold for these processes when we deal with the symbols of ordinary algebra.

* See Cayley's "Memoir on the Theory of Matrices," *Phil. Trans.*, 1858.