THE ADJUSTMENT AND COMPARISON OF OBSERVATIONS.

Lehrbuch der Ausgleichsrechnung nach der Methode der kleinsten Quadrate. Von Dr. K. J. BOBEK. Stuttgart, J. Maier, 1891. 8vo, pp. viii. + 176.

The Theory of Errors and Method of Least Squares. By WILLIAM WOOLSEY JOHNSON. New York, John Wiley & Sons, 1892. 12mo, pp. x. + 174.

THE text-book of Dr. Bobek is a curiosity as regards its catechetical form and typographical arrangement. In its substance, however, it is clear, sound, and highly practical. All the doubtful points regarding deductions of the law of facility of error, probable errors, and criteria for rejection of observations are passed over in silence, and the author in his 32 answers to questions, 29 explanations, and 52 problems writes with a certainty that should tend to inspire the student with confidence. Many lengthy examples of adjustments of observations in geodesy and physics are carried out in full detail with tabular forms for arranging the computations.

Professor Johnson's work is a careful and scholarly text-book on both the theory and practice of the subject. The doubtful points regarding the deduction of the law of error are not considered, and no place is given to criteria for rejection. More than one half the book, however, is devoted to discussions regarding the probability of errors, and to probable errors and comparisons of precision. The investigations regarding the probability surface and the probable errors of target shooting may be noted as one of these discussions, and another is that regarding the deductions of formulas for probable error. It is clearly pointed out that the probable error should be computed from the original individual observations and not from any weighted groups of these. It is shown that the assumption of the arithmetical mean involves the same law of errors for indirect as for direct observations. The way being thus made clear the whole discussion of indirect observations, both independent and conditioned, is given in the 29 pages of chapter VIII., while the last chapter treats of the solution of normal equations.

The tendency of modern text-books on the method of least squares in the direction of avoiding doubts regarding the deduction of the law of facility of error is clearly shown by the two works before us. The arithmetical mean being boldly assumed, and perhaps the assumption justified by a quotation from Gauss or by the discussion of Encke, the well-known demonstration follows, and the law of facility directly gives the conclusion that the most probable values of the observed quantities are those which render the sum of the squares of the residual errors a minimum, the precision of all the measurements being the same throughout. Some books, indeed, assume this final conclusion at the outset—a method probably more advantageous for certain classes of students than the procedure of Gauss's first proof. Gauss himself in later writings rejected this proof—partly, perhaps, on account of the assumption of the arithmetical mean as the most probable value, and partly, as Bertrand has suggested, on account of the insufficiency of the assumption that $y = \phi(x)$ represents the law of facility; for, the probability of an error depends both on its magnitude and on that of the measured quantity a, so that strictly the law is $y = \phi(a, x)$.

The idea of probable error is often slowly grasped by beginners. Professor Johnson's definition is an excellent one—"The error which is just as likely to be exceeded as not is called the probable error"—and the conception of regarding it as a measure of the risk of error will be of value to students. Nevertheless the tendency of text-books to devote a good deal of space to discussions and computations of probable error, mean error, and mean of all errors is in general to be deplored, as it is apt to convey an erroneous impression concerning the practical comparison of the precision of observations. An observer who thoroughly understands both his instrument and method of measurement is able to give a far better statement regarding the precision of the results than can any computer who deduces probable errors. The observer can with some confidence assign weights for the combination of measurements made at different times, but even he should hesitate to determine weights solely from computed probable errors. The method of least squares, in spite of the uncertainties of its fundamental proofs, is strong and sound when used for the adjustment of observations; notwithstanding the certainties of its reasoning it is often practically weak when used for the comparison of the precision of observations.

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